JULY 1957

# Journal
## of
# The Franklin Institute

1826

1957

**A Series of Experiments on Trees and Plants in Electrostatic Fields**
W. Wesley Hicks

**Additional Tables for Design of Optimum Ladder Networks**
Louis Weinberg

**Note on the Relativistic Harmonic Oscillator**
Louis Gold

**Errors in the Solution of Integral Equations**
Parry Moon and Domina Eberle Spencer

**Shear Buckling of Simply Supported Plates Tapered in Planform**
Bertram Klein

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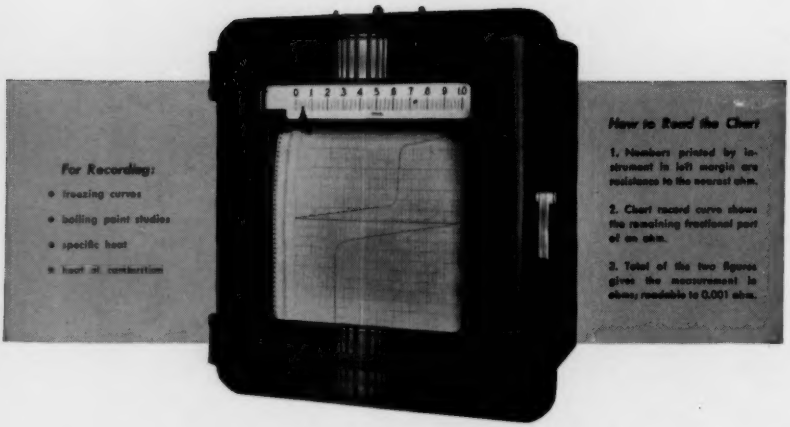
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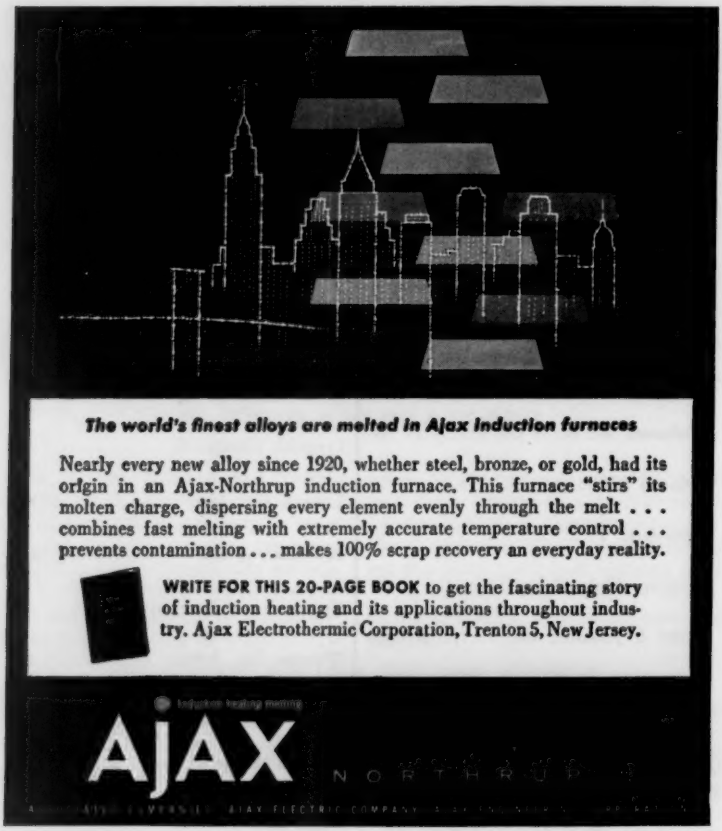
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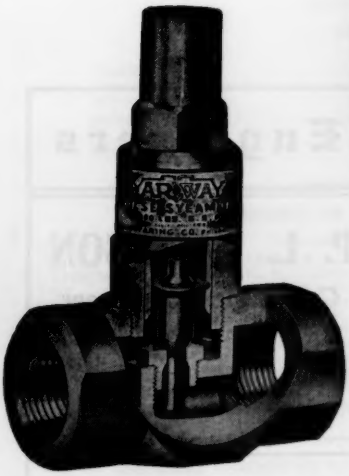

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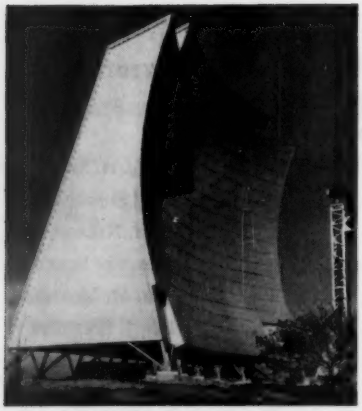


Giant over-the-horizon antenna designed by Bell Telephone Laboratories for "White Alice," Air Force Alaskan defense communications network.

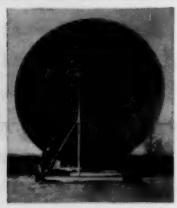
# *How UHF radio got seven-league boots*

THE huge antenna systems which project ultra-high frequency radio communications beyond the horizon began when a Bell Telephone Laboratories engineer became intrigued with a strange phenomenon. Although these radio waves were supposed to be useful only over line-of-sight distances, the waves displayed a mysterious tendency to take off in a giant stride to antennas beyond the horizon.

This phenomenon had been studied both here and abroad, but no practical use was seen until the engineer became interested and thoroughly sifted the experimental data. He came up with the stimulating conclusion that over-the-horizon transmission is far stronger and much more dependable than was generally supposed. Further he predicted that it could be utilized to supply dependable broadband communications. He and his associates at Bell Laboratories confirmed the prediction experimentally, then drew up requirements for the first over-the-horizon UHF transmission system.

This pioneer work at Bell Telephone Laboratories has greatly increased the usefulness of UHF communications. For example, over-the-horizon transmission now provides critically important communications between remote military outposts in the Arctic and in the far north.

For the Bell System it can provide important new links for telephone conversations and television.



Left, Kenneth Bullington, B.S.E.E., University of New Mexico; M.S., Massachusetts Institute of Technology; recipient of the 1956 Morris Liebmann Memorial Prize and the 1956 Stuart Ballantine Medal for his contributions in the field of over-the-horizon ultra-high frequency radio transmission.

Right, experimental antenna used in early over-the-horizon UHF radio transmission research. Research extended transmission reach from 30 miles line-of-sight to 200 miles.

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Vol. 264 JULY, 1957 No. 1

# A SERIES OF EXPERIMENTS ON TREES AND PLANTS IN ELECTROSTATIC FIELDS *

BY

W. WESLEY HICKS [1]

Many investigators have been intrigued by the possibility that electrostatic fields have an influence of some kind upon plant material. Recent research into the effect of ionized air upon animal cells has brought to light new reasons to suspect that a more or less parallel situation may prevail in the field of botany.

Such is the background of the series of home experiments here described. Their object, it is needless to say, was not to obtain quantitative data as to responses, nor even to establish any consistent direction of change. Cultural conditions were, however, kept as nearly uniform as possible, with techniques along several different lines that anyone who is interested can readily undertake in his own greenhouse or garden.

The present paper gathers together a number of simple procedures, including some that are original, and reports certain findings which appear suggestive. The hope is that sufficient interest may be aroused to lead to more organized research by those whose training and facilities permit a critical investigation of the subject.

## A. PLANTS WITHIN WIRE-SCREEN CAGES

### *Pelargoniums*

In the experiment reported here, each plant was surrounded by a wire-screen cage of four-to-the-inch mesh, cylindrical in three cases and rectangular in the fourth.

* This study is a by-product of a more important, completely controlled investigation into the effect of ion-augmented electrostatic fields on human beings and animals, currently being reported in scientific and medical journals. This work was summarized in W. Wesley Hicks, "Air Ion Generation, Separation, Metering and Physiological Effects," JOUR. FRANKLIN INST., Vol. 261, p. 209 (1956).

[1] President, Wesix, San Francisco, Calif.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.

Four pelargoniums of the same height, 9½ in., were planted 2 ft. apart—one surrounded by a 12 × 14½-in. cage suspended by nylon string a few inches above the ground, one with a like cage resting on the ground, and one (for control) left without a cage. The fourth plant, which was kept within an open Hertz ring, is reported upon later in this paper.

Table I shows the increase in height and in number of flower clusters after 36, 49 and 96 days.

TABLE I.

| Days After Start of Experiment | Suspended Insulated Cage | | Grounded Cage | | Control | |
|---|---|---|---|---|---|---|
| | Height, in. | Bloom Clusters | Height, in. | Bloom Clusters | Height, in. | Bloom Clusters |
| 0 | 9½ | – | 9½ | – | 9½ | – |
| 36 | 11½ | – | 10½ | – | 9½ | – |
| 49 | 14½ | – | 13½ | – | 11 | – |
| 96 | 15 | 7 | 13½ | 4 | 12 | 6 |

The preceding test was repeated with ten pelargoniums 10 in. high. Eight plants were surrounded with 12 × 14-in. screen cages. Of these, four carried an artificial charge with a potential of 120 volts to ground, two cages with a positive charge and two negative. Two other cages were grounded, and the remaining two plants, without cages, served as controls. Plants in the positively-charged cages grew faster and bloomed two weeks earlier than those in the negatively-charged cages; the latter, in turn, grew faster and bloomed earlier than the controls. The two plants in the grounded cages died.

*Rhododendrons*

Two rhododendrons were surrounded with wire-screen cages measuring 24 × 36 in., insulated from the ground. One cage carried a positive charge, the other a negative charge (both supplied through rectifiers from an a-c. source, with 120 volts to ground). The third plant, without a cage, served as a control. When the experiment began, each plant had eight branches and eight healthy buds.

In the negative cage each of the buds developed into flowers. In the positively-charged cage three buds developed as flowers, and the other five into leaves and stems. Gains in height were 3½ to 5½ in. in the negative cage, and 9½ to 11½ in. more in the positive cage. Three of the branches in the negatively-charged cage died, whereas all eight branches in the positively-charged cage showed healthy growth. During the test period the control plant developed only one bloom. Two years after the cages were dismantled, the plant from the positive cage still continued to show the greatest growth.

*Beans*

Two lots of beans were planted in June, one surrounded by a rectangular suspended cage with a positive charge and a potential of 120 volts to ground, and the other lot without a cage. After 75 days the plants in the positively-charged cage had grown 8 ft. 3 in., while those without a cage had grown 4 ft.

*Holly Trees*

Three English holly trees were set out in January, one surrounded by a suspended cage without artificial charge, one in a grounded cage, and one (for control) without the cage. By March the plant in the suspended cage had grown 6 in. taller than that in the grounded cage, and 3 in. more than the control.

*Summary*

Vegetative growth of pelargoniums, rhododendrons, beans, and holly trees was stimulated by electric fields. It was greatest in the positive fields, next greatest in negative fields, less for the controls, and least in the grounded cages. There was evidence that the stronger the field the greater the growth.

### B. PLANTS EQUIPPED WITH ANTENNAS

*Cherry Trees*

In October, two cherry trees 50 in. high were set out 15 ft. apart. One was equipped with a 60-in. antenna, lightly clamped to the trunk of the tree and extending 36 in. above the top. The second tree without antenna served as a control. After six months, the tree with the antenna developed 24 blossoms 21 days before any flowers appeared on the control tree, although the latter eventually developed the same number of blossoms. However, no fruit formed on the test tree, whereas all blooms on the control matured into fruit.

Five months later, the control tree had put out noticeably the greater leaf growth. By the following spring, the tree with the antenna had six blossoms with 60 leaf clusters, while the control tree had 36 blossoms with 60 leaf clusters. All the blossoms on the control tree were followed by fruit, while no cherries developed on the test tree.

One year later, the growth of the tree with the antenna appeared arrested, and the control tree had gained 2 ft. in height.

*Avocado Trees*

A 5-ft. avocado tree, with ten branches, was equipped with a $\frac{1}{8}$-in. wire antenna, 60 in. long, lightly strapped against the trunk of the tree

and extending 30 in. above the highest branch. In addition, each limb was wrapped loosely with a 14-gauge copper-wire antenna terminating on the vertical antenna. Each lateral antenna extended 6 in. beyond the tip of its branch.

After four months, a branch which left the trunk just below the lower end of the vertical antenna had outgrown the main trunk by 30 in. Obviously the trunk was being hurt by the antenna, which was then removed. Within the same period leaves along each branch had dropped off, leaving only new growth at the tips which had grown several inches. After the lateral antennas were all removed, leaves again developed along the branches.

In this tree, the capillary roots were extended with 36-gauge nickel chromium wire to a depth of 4 ft. below the surface, and the tree appeared to thrive with a minimum amount of water through a dry summer.

In a second larger avocado tree, which bore a dozen fruits, an antenna composed of a 12-in. cylinder of wire-screen material was placed in the center of the tree extending 2 ft. above its highest branches but carefully insulated from all contact with the tree or its leaves. Growth was stimulated to such an extent that only one fruit developed while every branch of the tree showed vigorous growth and bloom. Top branches grew two feet in a few months.

### C. PLANTS WITHIN HERTZ RINGS

*Camellias*

Two camellia seedlings 3 in. high, each having a single pair of leaves, were planted outdoors under like conditions on the same day. Over one plant was placed an assembly of nine Hertz open copper rings spaced 1 in. apart. The smallest ring was 2 in. in diameter and the largest 18 in. Rings were evenly spaced one within the other and tied together with nylon string, simulating a Lakhovsky Multiple Wave Length Resonator, or Receptor, but supported horizontally instead of vertically. The theoretical wave length to which the rings oscillate is 0.5 to 5 meters. Four 1-in. wooden posts and four nylon strings about 30 in. long supported the assembly, which took the shape of a flat cone with the smallest ring at the top.

This multiple-ring arrangement was purposely adopted to minimize the effect of the electrical gradient and the collection of atmospheric electricity from lateral air movement, as well as to avoid interference with sunlight. Table II shows the increase in height and number of branches in 9, 15, 21 and 25 months. Continuing tests show similar results.

*Geraniums*

Of two similar geraniums planted at the same time, one which was within an 11½-in. open ring grew noticeably faster than the control plant near it with no ring, and bloomed two weeks sooner. An English holly seedling just outside of the ring outgrew other seedlings of the same age.

TABLE II.

| Months after Start of Experiment | Test Plant | | Control | |
|---|---|---|---|---|
| | Height, in. | Number of Branches | Height, in. | Number of Branches |
| 0 | 3 | 1 | 3 | 1 |
| 9 | 4 | 1 | 4 | 1 |
| 15 | 5 | 4 | 5 | 2 |
| 21 | 6½ | 9 | 6½ | 2 |
| 25 | 10½ | 37 | 6½ | 3 |

*Mistletoe*

A small clump of mistletoe on a nut tree was surrounded with a 4-in. open copper-wire ring insulated from the tree. Table III shows the number of branches and the growth after 8, 15 and 21 months.

TABLE III.

| Months after Start of Experiment | Cluster within Copper Ring | | Control Clusters on Neighboring Trees (Average size, in.) |
|---|---|---|---|
| | Number of Branches | Length of Longest Branch, in. | |
| 0 | 1 | 1½ | 3× 5 |
| 8 | 1 | 2½ | 12×12 |
| 15 | 3 | 3 | 13×18 |
| 21 | 8 | 6 | 24×36 |

At the end of the experiment, every shoot and leaf on the limb with the ringed mistletoe had died, leaving the limb and all its side branches barren for a distance of 12 in. under the ring, 18 in. each side of it and 24 in. above it.

**Electronic Time of Flight Analyzer.**—Two General Electric Company engineers at the Knolls Atomic Power Laboratory have developed a new electronic computing device for measuring, counting and classifying time of flight of neutrons from a nuclear reactor which accomplishes in an hour what it previously would have taken a man a month to do.

Elmer J. Wade and Donald S. Davidson of KAPL developed the first all-transistorized time of flight analyzer for use in determining the neutron energy distribution of various types of reactors. It contains 256 channels, compared to a device previously in use at KAPL which furnished only 11 channels for recording information on different energy range neutrons.

Employed in reactor development work at KAPL, the analyzer, through its work-saving features, has freed highly trained scientists from routine calculating work. They can now spend time saved in evaluating information the new analyzer provides for them.

Savings in time and labor by the new analyzer are matched by the space it conserves at the laboratory, operated by General Electric for the Atomic Energy Commission. The device is only 8 inches high, 19 inches long and 13 inches deep, compared to earlier analyzer equipment which took up at least 10 times that amount of space at the laboratory. Most of the space saving came in use of 293 transistors in place of as many vacuum tubes, along with more compact circuitry through use of printed circuits. In addition, the new analyzer uses only eight watts of power, compared to two kilowatts needed to operate the earlier vacuum tube equipped analyzer.

The KAPL engineers said the analyzer works this way:

Development of new reactors at KAPL involves mock-up of various fuel arrangements and accurate measurement of resulting neutron energy spectrum.

This is accomplished in a large part by measuring the time of flight of neutrons over a given distance and recording the number of these neutrons which fall into 256 different velocity ranges. Through various calculations, scientists can convert the velocity of these neutrons into corresponding energy ranges.

To accumulate this vital information, a neutron beam coming out of the thermal test reactor is chopped by a rotating shutter so that velocity can be measured. At the end of a flight path of established length, the neutrons are detected by a counter and the information relayed to the analyzer.

Information on each neutron counted is directed into one of 256 channels, each representing a velocity range. Up to 65,536 neutron counts are stored in each of these channels.

After the experiment this information is transmitted over cables to electronic calculating equipment which punches information from each channel of the analyzer onto a separate card.

This information is then fed into a digital computer for analysis of the data from the cards and the result is then automatically plotted on a graph which shows the reactor energy characteristics.

The entire operation from neutron count to production of a graph which reveals the energy spectra of the reactor requires only about an hour, compared to what would have been a month's work for a single scientist recording data manually.

# ADDITIONAL TABLES FOR DESIGN OF OPTIMUM LADDER NETWORKS *

BY

LOUIS WEINBERG [1]

Part I **

ABSTRACT

In a preceding paper tables were presented for the design of three large classes of ladder networks. These networks had characteristics given by Butterworth, Tschebyscheff, and Bessel polynomials. In the present paper a number of additional tables are presented, the tables now being classified on the basis of the parameter $r$, which is the input-to-output resistance or conductance ratio. The tables give the element values of normalized low-pass ladders with one of the following characteristics: maximally flat magnitude (Butterworth), equal-ripple magnitude (Tschebyscheff), and maximally flat time delay (Bessel polynomial). By means of frequency transformations the networks given by the tabulated element values for the Butterworth and Tschebyscheff networks may be converted to give high-pass, band-pass, and band-elimination filters. Thus the tables may be used as a handbook for the design of these optimum networks.

## INTRODUCTION

Tables for the design of three large classes of networks were presented in a preceding paper (1).[2] It was shown there that by use of these tables the engineer who knows little about the theory of modern synthesis can synthesize useful networks. These networks had characteristics given by Butterworth, Tschebyscheff, and Bessel polynomials.

In this paper the same characteristics are considered. However, many new tables are added. In addition, the basis for classifying the tables has been changed to one that is believed to be more useful for most applications.

The following extensions and changes have been made:

1. The tables are no longer classified in terms of the decrement ratio $D$, as they were in the preceding paper. The basis for classification in this paper is $r$, where $r = R_n/R_1$ (or $r = G'_n/G'_1$), the ratio of the input to the output resistance (or input-to-output conductance). These tables give a much larger range of input-to-output terminations than was available in the previously published tables.

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* This paper is based on the author's report with the same title, Technical Memorandum No. 434, Hughes Research Laboratories, Culver City, Calif.

[1] Research Laboratories, Hughes Aircraft Co., Culver City, Calif.

** Part II will appear in this JOURNAL for August, 1957.

[2] The boldface numbers in parentheses refer to the references appended to Part I of this paper.

2. Tables for the Tschebyscheff characteristic with $\frac{1}{10}$-db and $\frac{1}{4}$-db. ripple have been added.

3. In the preceding paper the Bessel-polynomial networks were given only for a resistance termination at one end. In this paper the networks are given for the same wide range of resistance ratios as are the Butterworth and Tschebyscheff networks.

It is felt that since these tables eliminate tedious computations, they will be of great value to engineers familiar with synthesis theory. But the paper is directed mainly at those practical engineers who know little about synthesis and desire a final working formula and a set of tabulated values. It was therefore decided to make the paper as self-contained as possible, within the space restrictions, by giving an analytical discussion of each type of network. Though the discussion is necessarily brief, it is believed that it is sufficient to acquaint the reader with the characteristics of the networks whose element values are given in the tables.

As mentioned in Sec. V, a number of different networks is possible, each of which realizes the *identical* transfer function (including the constant multiplier). The networks differ because of the different choices for the zeros of the reflection coefficient. The tables in this paper are based on the choice that gives maximum gain-bandwidth product for a specified value of shunt capacitance (**2**), that is, all the zeros are chosen to lie in one half-plane.

The paper is divided into five main sections. How to use the tables of element values is discussed in the first section. The next two treat Butterworth and Tschebyscheff networks, respectively, while the fourth section treats the maximally flat time-delay networks obtained by the use of Bessel polynomials. In each of these three sections tables of the element values of the normalized low-pass ladder network are given. In the final section it is briefly shown: (*a*) how to transform Butterworth and Tschebyscheff networks to serve high-pass, band-pass, or band-elimination functions; (*b*) how to remove the normalization of the element values—that is, how to change the pass band of the network from $\omega = 1$ to the desired radian frequency, and how to raise the level of the network; (*c*) how to use duality and reciprocity to obtain sets of new networks; and (*d*) how to convert the symmetrical Butterworth and Tschebyscheff networks to unsymmetrical ones with any desired ratio of input-to-output resistance.

## I. USE OF THE TABLES OF ELEMENT VALUES

The general form of the low-pass ladder network whose element values are given in the tables is a lossless network terminated in resistance. In all the tables and in the figures the element values are in ohms, henrys, and farads. The tabulated element values are normalized in that the pass band has a cutoff radian frequency $\omega_c$ equal to

unity and the network has a one-ohm resistance load, that is, in all the tables $R_1 = 1$. As shown in Sec. V and in the illustrative examples the removal of these normalizations requires only simple multiplications.

Six different values of $r = R_n/R_1$ (or $r = G'_n/G'_1$) are included: 0, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, and 1. Since $R_1 = 1$ for all the tables, the value of $R_n$ for each table is given by the $r$ of that table and thus need not be tabulated.

The primed and the unprimed values in the tables yield dual networks so that a transfer impedance or a transfer admittance can be realized. The networks for the general ladder, with a resistance termination at both ends and with a current-source input, are shown in Fig. 1 for $n$ odd and in Fig. 2 for $n$ even; $n$ is the degree of the denominator of the transfer function and is thus also equal to the number of reactances in the ladder. The transfer function realized by these networks is the transfer impedance $Z_{21} = E_2/I_1$.

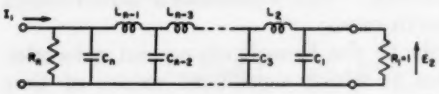

FIG. 1. General form of low-pass ladder network with a current-source input and $n$ odd.

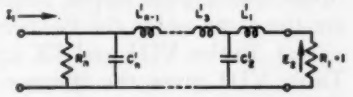

FIG. 2. General form of low-pass ladder network with a current-source input and $n$ even.

For a voltage source used as the input, the dual of the above networks can be used. The networks are shown in Figs. 3 and 4 for $n$ odd and even, respectively. The transfer function realized by these networks is the transfer admittance $Y_{21} = I_2/E_1$.

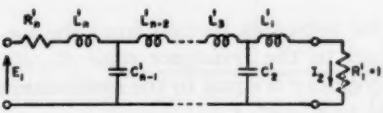

FIG. 3. General form of low-pass ladder network with a voltage-source input and $n$ odd.

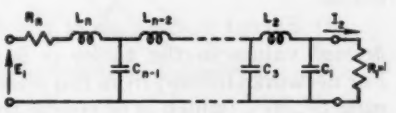

FIG. 4. General form of low-pass ladder network with a voltage-source input and $n$ even.

It is also pointed out that Thévenin's or Norton's theorem can be used to effect a source conversion and thus yield new network configurations. For example, Thévenin's theorem applied to $R_n$ and the current source in Fig. 1 yields a voltage source and a series resistance. In this way a transfer admittance or transfer voltage ratio may be realized with a shunt capacitance branch at both ends of the coupling network.

The primed and unprimed elements shown in Figs. 1–4 are intended to correspond to the primed and unprimed values given in the tables.

Thus for a specified $Z_{21}$ and *n odd* the *unprimed* tabulated values yield the network of Fig. 1, whereas for *n even* the *primed* tabulated values yield the network of Fig. 2. When a transfer admittance $Y_{21}$ is required, the *primed* tabulated values yield the network shown in Fig. 3 for *n odd*, and for *n even* the *unprimed* values yield the network of Fig. 4.

The tables are divided as follows:

(*a*) Table I gives the element values for the Butterworth filter, where (*a*) applies for $r = 0$, (*b*) for $r = \frac{1}{8}$, (*c*) for $r = \frac{1}{4}$, (*d*) for $r = \frac{1}{3}$, (*e*) for $r = \frac{1}{2}$, and (*f*) for $r = 1$.

(*b*) Tables II–VII apply to the Tschebyscheff filter, with the subdivisions being necessary to provide for the different ripple factors. For example, Tables II and III give the element values for a $\frac{1}{10}$-db ripple and a $\frac{1}{4}$-db ripple, respectively. The alphabetical subdivisions are the same as for the Butterworth case.

(*c*) Tables VIII and IX apply to the Bessel-polynomial networks. Table VIII gives the frequencies at which significant values of time delay and loss occur. The variable $u$ is the normalized frequency $\omega/\omega_0$, where $\omega_0 = 1/t_0$ and $t_0$ is the desired time delay, that is, the time delay occurring at zero frequency. Table IX gives the element values; the alphabetical subdivisions are the same as for the Butterworth and Tschebyscheff cases.

As has been mentioned, the parameter $r$ is equal either to the ratio of the input to the output *resistance* or to the input-to-output *conductance* ratio. A little reasoning always suffices to determine which applies, but for convenience the practical rules are expressed explicitly below.

(*a*) Except in the case of $r = 0$, for networks formed from the *unprimed* values in the tables, $r$ is equal to the *resistance ratio* $R_n/R_1$. For networks formed from the *primed* values $r$ is equal to the *conductance ratio* $G'_n/G'_1$ (which is of course equal to $R'_1/R'_n$).

(*b*) For $r = 0$ the parameter $r$ is equal to the *resistance ratio* for the combinations: (1) *unprimed* values and *n even*, and (2) *primed* values and *n odd*. It is equal to the *conductance ratio* for (1) *unprimed* values and *n odd*, and (2) *primed* values and *n even*. This is merely a detailed way of stating that for $r = 0$, $R_n$ *becomes a short* for networks with a *series input*, that is, those in Figs. 3 and 4, and $R'_n$ *becomes an open circuit* for networks with a *shunt input*, that is, those in Figs. 1 and 2.

Inspection of the tables for the Tschebyscheff networks shows that for a number of tables element values are not given for $n$ even. For these cases the specified resistance (or conductance) ratio is too large to be physically realizable. This occurs for:

(*a*) $r = 1$ for all ripples.
(*b*) $r = \frac{1}{2}$ for 1-db, 2-db, and 3-db ripples.
(*c*) $r = \frac{1}{3}$ for 2-db and 3-db ripples.
(*d*) $r = \frac{1}{4}$ for 2-db and 3-db ripples.

With the above preliminary remarks the steps in the procedure for using the tables follow:

1. Determine from the specifications of the problem whether a Butterworth, Tschebyscheff, or Bessel-polynomial network is to be used.
2. Calculate the value of $n$ that gives the required degree of the denominator polynomial of the transfer function and consequently the required complexity of the network. For the Tschebyscheff characteristic it is first necessary to calculate the ripple factor $\epsilon$.
3. Using this value of $n$ look up the element values in the appropriate table.
4. Remove the normalizations as shown in Sec. IV. The bandwidth is thus changed from $\omega_c = 1$ to the desired cutoff value, and the load resistance and the network level are changed to the required values.

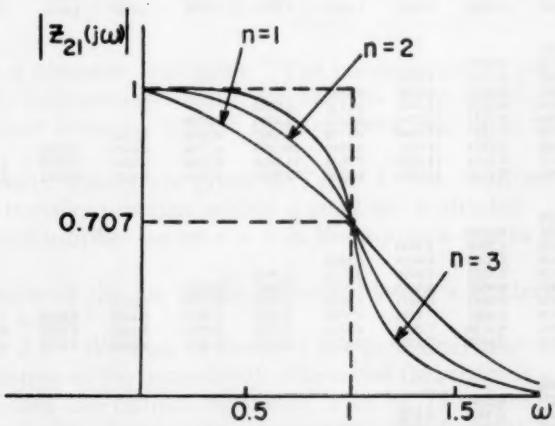


FIG. 5. Sketches of the first three orders of the Butterworth approximation to the low-pass filter.

5. If a high-pass, band-pass, or band-elimination network is desired, convert the element values by means of the frequency transformations of Sec. V.

In carrying out the first step, if we require a filter whose magnitude characteristic is specified, then the Butterworth or Tschebyscheff characteristic may be used. For the same value of $n$, the Tschebyscheff filter gives a better coverage of the pass band and a faster drop-off outside the band than any other possible transfer function that is also a constant divided by a polynomial; its phase characteristic, however, is more nonlinear than that of the Butterworth filter. Thus the choice between the two will depend on the importance of the phase characteristic. Neither of these filters gives a linear phase characteristic, that is, a pure time delay, over a specified frequency range. For this purpose

the Bessel polynomials are used. The filters obtained by use of the Bessel polynomials also have a low-pass magnitude characteristic so that they may also be used in those problems where the magnitude characteristic is specified.

The method for calculating the value of $n$ and, in the case of the Tschebyscheff filter, the value of $\epsilon$, is shown in the respective sections.

TABLE I.—*Element Values (in ohms, henrys, farads) for a Normalized Butterworth Filter.*

| Value of n | $C_1$ or $L_1'$ | $L_2$ or $C_2'$ | $C_3$ or $L_3'$ | $L_4$ or $C_4'$ | $C_5$ or $L_5'$ | $L_6$ or $C_6'$ | $C_7$ or $L_7'$ | $L_8$ or $C_8'$ | $C_9$ or $L_9'$ | $L_{10}$ or $C_{10}'$ |
|---|---|---|---|---|---|---|---|---|---|---|
| a) r = 0 | | | | | | | | | | |
| 1 | 1.0000 | | | | | | | | | |
| 2 | 0.7071 | 1.4142 | | | | | | | | |
| 3 | 0.5000 | 1.3333 | 1.5000 | | | | | | | |
| 4 | 0.3827 | 1.0824 | 1.5772 | 1.5307 | | | | | | |
| 5 | 0.3090 | 0.8944 | 1.3820 | 1.6944 | 1.5451 | | | | | |
| 6 | 0.2588 | 0.7579 | 1.2016 | 1.5529 | 1.7593 | 1.5529 | | | | |
| 7 | 0.2225 | 0.6560 | 1.0550 | 1.3972 | 1.6588 | 1.7988 | 1.5576 | | | |
| 8 | 0.1951 | 0.5776 | 0.9370 | 1.2588 | 1.5283 | 1.7287 | 1.8246 | 1.5607 | | |
| 9 | 0.1736 | 0.5155 | 0.8414 | 1.1408 | 1.4037 | 1.6202 | 1.7772 | 1.8424 | 1.5628 | |
| 10 | 0.1564 | 0.4654 | 0.7626 | 1.0406 | 1.2921 | 1.5100 | 1.6869 | 1.8121 | 1.8552 | 1.5643 |
| b) r = 1/8 | | | | | | | | | | |
| 1 | 9.0000 | | | | | | | | | |
| 2 | 11.9764 | 0.0939 | | | | | | | | |
| 3 | 12.4442 | 0.1735 | 4.1674 | | | | | | | |
| 4 | 12.5685 | 0.2032 | 8.9296 | 0.0493 | | | | | | |
| 5 | 12.6076 | 0.2169 | 11.3305 | 0.1146 | 2.5343 | | | | | |
| 6 | 12.6190 | 0.2243 | 12.6794 | 0.1533 | 6.1898 | 0.0330 | | | | |
| 7 | 12.6199 | 0.2287 | 13.5040 | 0.1778 | 8.5907 | 0.0835 | 1.8121 | | | |
| 8 | 12.6166 | 0.2314 | 14.0417 | 0.1940 | 10.2279 | 0.1190 | 4.6929 | 0.0248 | | |
| 9 | 12.6117 | 0.2333 | 14.4102 | 0.2053 | 11.3856 | 0.1446 | 6.8248 | 0.0653 | 1.4086 | |
| 10 | 12.6064 | 0.2346 | 14.6730 | 0.2135 | 12.2305 | 0.1635 | 8.4293 | 0.0965 | 3.7699 | 0.0198 |
| c) r = 1/4 | | | | | | | | | | |
| 1 | 5.0000 | | | | | | | | | |
| 2 | 6.2741 | 0.1992 | | | | | | | | |
| 3 | 6.3870 | 0.3608 | 2.1699 | | | | | | | |
| 4 | 6.3840 | 0.4180 | 4.6024 | 0.1018 | | | | | | |
| 5 | 6.3636 | 0.4435 | 5.8036 | 0.2350 | 1.2992 | | | | | |
| 6 | 6.3425 | 0.4567 | 6.4673 | 0.3130 | 3.1601 | 0.0675 | | | | |
| 7 | 6.3238 | 0.4641 | 6.8671 | 0.3618 | 4.3727 | 0.1700 | 0.9225 | | | |
| 8 | 6.3078 | 0.4687 | 7.1244 | 0.3940 | 5.1943 | 0.2417 | 2.3838 | 0.0503 | | |
| 9 | 6.2941 | 0.4716 | 7.2984 | 0.4162 | 5.7720 | 0.2932 | 3.4607 | 0.1325 | 0.7143 | |
| 10 | 6.2825 | 0.4735 | 7.4209 | 0.4321 | 6.1916 | 0.3312 | 4.2683 | 0.1955 | 1.9091 | 0.0401 |
| d) r = 1/3 | | | | | | | | | | |
| 1 | 4.0000 | | | | | | | | | |
| 2 | 4.8284 | 0.2761 | | | | | | | | |
| 3 | 4.8473 | 0.4934 | 1.6725 | | | | | | | |
| 4 | 4.8105 | 0.5676 | 3.5233 | 0.1386 | | | | | | |
| 5 | 4.7743 | 0.5997 | 4.4239 | 0.3186 | 0.9912 | | | | | |
| 6 | 4.7446 | 0.6156 | 4.9155 | 0.4233 | 2.4042 | 0.0913 | | | | |
| 7 | 4.7206 | 0.6244 | 5.2085 | 0.4882 | 3.3200 | 0.2294 | 0.7006 | | | |
| 8 | 4.7012 | 0.6295 | 5.3950 | 0.5308 | 3.9376 | 0.3258 | 1.8075 | 0.0678 | | |
| 9 | 4.6853 | 0.6326 | 5.5200 | 0.5601 | 4.3702 | 0.3948 | 2.6209 | 0.1785 | 0.5410 | |
| 10 | 4.6720 | 0.6346 | 5.6071 | 0.5809 | 4.6833 | 0.4454 | 3.2293 | 0.2630 | 1.4445 | 0.0540 |
| e) r = 1/2 | | | | | | | | | | |
| 1 | 3.0000 | | | | | | | | | |
| 2 | 3.3461 | 0.4483 | | | | | | | | |
| 3 | 3.2612 | 0.7789 | 1.1811 | | | | | | | |
| 4 | 3.1868 | 0.8826 | 2.4524 | 0.2175 | | | | | | |
| 5 | 3.1331 | 0.9237 | 3.0510 | 0.4955 | 0.6857 | | | | | |
| 6 | 3.0938 | 0.9423 | 3.3687 | 0.6542 | 1.6531 | 0.1412 | | | | |
| 7 | 3.0640 | 0.9513 | 3.5532 | 0.7512 | 2.2726 | 0.3536 | 0.4799 | | | |
| 8 | 3.0408 | 0.9558 | 3.6678 | 0.8139 | 2.6863 | 0.5003 | 1.2341 | 0.1042 | | |
| 9 | 3.0223 | 0.9579 | 3.7436 | 0.8565 | 2.9734 | 0.6046 | 1.7846 | 0.2735 | 0.3685 | |
| 10 | 3.0072 | 0.9588 | 3.7934 | 0.8864 | 3.1795 | 0.6808 | 2.1943 | 0.4021 | 0.9818 | 0.0825 |
| f) r = 1 | | | | | | | | | | |
| 1 | 2.0000 | | | | | | | | | |
| 2 | 1.4142 | 1.4142 | | | | | | | | |
| 3 | 1.0000 | 2.0000 | 1.0000 | | | | | | | |
| 4 | 0.7654 | 1.8478 | 1.8478 | 0.7654 | | | | | | |
| 5 | 0.6180 | 1.6180 | 2.0000 | 1.6180 | 0.6180 | | | | | |
| 6 | 0.5176 | 1.4142 | 1.9319 | 1.9319 | 1.4142 | 0.5176 | | | | |
| 7 | 0.4450 | 1.2470 | 1.8019 | 2.0000 | 1.8019 | 1.2470 | 0.4450 | | | |
| 8 | 0.3902 | 1.1111 | 1.6629 | 1.9616 | 1.9616 | 1.6629 | 1.1111 | 0.3902 | | |
| 9 | 0.3473 | 1.0000 | 1.5321 | 1.8794 | 2.0000 | 1.8794 | 1.5321 | 1.0000 | 0.3473 | |
| 10 | 0.3129 | 0.9080 | 1.4142 | 1.7820 | 1.9754 | 1.9754 | 1.7820 | 1.4142 | 0.9080 | 0.3129 |

### II. BUTTERWORTH CHARACTERISTIC (3,4,5)

The Butterworth function is used to approximate the squared magnitude of a transfer function. For the transfer impedance it is given by

$$|Z_{21}(j\omega)|^2 = \frac{1}{1 + \omega^{2n}}. \tag{1}$$

This function gives an approximation to a low-pass filter characteristic; sketches of the Butterworth approximation for the first three values of $n$ are shown in Fig. 5. The Butterworth function is said to have a *maximally flat* magnitude characteristic.

By use of Eq. 1 the complete transfer function is given as

$$Z_{21}(s) = \frac{H}{B_n(s)}, \tag{2}$$

where $H$ is a constant multiplier. The polynomials $B_n$ are called the Butterworth polynomials; these polynomials have a unity coefficient for $s^n$ and their zeros are the $n$th roots of unity that lie in the left half-plane.

The element values are given in Table I; the resulting networks realize the transfer function within a constant multiplier. To obtain the constant multiplier we let $s = 0$ in the network and in the transfer function.

An example of the use of the tables to design a Butterworth filter is presented below.

*Example 2.1.* We wish to design a low-pass filter that has a resistance termination at the output only. The cutoff frequency is $\omega_c = 10{,}000$ radians/sec and the output resistance is to be 750 ohms. At a frequency $\omega = 3\omega_c$ the magnitude response is to be down at least 50 db. The input source is a cathode follower which approximates a true voltage source.

First we determine the value of $n$.

$$\left.\frac{1}{1 + \omega^{2n}}\right|_{\omega=3} = 10^{-5}$$

$$(1 + \omega^{2n})|_{\omega=3} = 10^5$$

$$3^{2n} \cong 10^5$$

$$n = \frac{1}{2}\frac{5}{\log 3}$$

$$= 5.23.$$

The next larger integer $n = 6$ must be used.

Since no input resistance is required, the table for $r = 0$ is used, namely, Table I($a$). Since the input is a voltage source and $n$ is even, the network form of Fig. 4 (with $R_n$ omitted) is applicable; that is, the unprimed element parameters are used.

Consulting the tables yields the element values

$$R_1 = 1 \qquad L_4 = 1.553$$
$$C_1 = 0.2588 \qquad C_5 = 1.759$$
$$L_2 = 0.7579 \qquad L_6 = 1.553$$
$$C_3 = 1.202$$

To obtain a load resistance of 750 ohms, we multiply $R_1$ and all $L$'s and divide all $C$'s by 750. To change the cutoff frequency to 10,000 rad/sec every $L$ and $C$ must be divided by this value.

The final values are therefore

$$R = 750\,R_1 = 750 \qquad L_d = \frac{RL_4}{\omega_c} = 1.16 \times 10^{-1}$$
$$C_a = \frac{C_1}{\omega_c R} = 3.45 \times 10^{-8} \qquad C_e = \frac{C_5}{R\omega_c} = 2.34 \times 10^{-7}$$
$$L_b = \frac{RL_2}{\omega_c} = 5.68 \times 10^{-2} \qquad L_f = \frac{RL_6}{\omega_c} = 1.16 \times 10^{-1}$$
$$C_c = \frac{C_3}{\omega_c R} = 1.60 \times 10^{-7}$$

and the network is shown in Fig. 6.

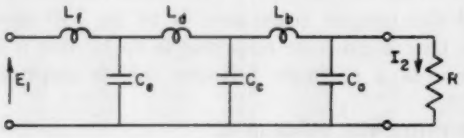


FIG. 6. Final network achieved for Example 2.1.

At $s = 0$ the network becomes a pure resistance and therefore

$$Z_{21}|_{s=0} = \frac{H}{B_6(0)} = R = 750.$$

Since the constant term of every $B_n$ is unity, the constant multiplier $H$ is 750. The transfer voltage ratio $E_2/E_1$, since $E_2 = 750\,I_2$, is given by

$$\frac{E_2}{E_1} = \frac{1}{B_6(s)}.$$

### III. TSCHEBYSCHEFF CHARACTERISTIC (6)

The Tschebyscheff approximation to the magnitude characteristic of a low-pass filter is given by

$$|Z_{21}(j\omega)|^2 = \frac{1}{1 + [\epsilon T_n(\omega)]^2}. \tag{3}$$

TABLE II.—*Element Values (in ohms, henrys, farads) for a Normalized Tschebyscheff Filter with $\frac{1}{10}$-db Ripple ($\epsilon = 0.1526$, $\epsilon^2 = 0.0233$).*

| Value of n | $C_1$ or $L_1'$ | $L_2$ or $C_2'$ | $C_3$ or $L_3'$ | $L_4$ or $C_4'$ | $C_5$ or $L_5'$ | $L_6$ or $C_6'$ | $C_7$ or $L_7'$ | $L_8$ or $C_8'$ | $C_9$ or $L_9'$ | $L_{10}$ or $C_{10}'$ |
|---|---|---|---|---|---|---|---|---|---|---|
| a) r = 0 | | | | | | | | | | |
| 1 | 0.1526 | | | | | | | | | |
| 2 | 0.4215 | 0.7159 | | | | | | | | |
| 3 | 0.5158 | 1.0864 | 1.0895 | | | | | | | |
| 4 | 0.5544 | 1.1994 | 1.4576 | 1.2453 | | | | | | |
| 5 | 0.5734 | 1.2490 | 1.5562 | 1.5924 | 1.3759 | | | | | |
| 6 | 0.5841 | 1.2752 | 1.5999 | 1.6749 | 1.7236 | 1.4035 | | | | |
| 7 | 0.5906 | 1.2908 | 1.6236 | 1.7107 | 1.7987 | 1.7395 | 1.4745 | | | |
| 8 | 0.5949 | 1.3008 | 1.6380 | 1.7302 | 1.8302 | 1.8070 | 1.8163 | 1.4660 | | |
| 9 | 0.5978 | 1.3076 | 1.6476 | 1.7423 | 1.8473 | 1.8343 | 1.8814 | 1.7991 | 1.5182 | |
| 10 | 0.6000 | 1.3124 | 1.6542 | 1.7503 | 1.8579 | 1.8489 | 1.9068 | 1.8600 | 1.8585 | 1.4964 |
| b) r = 1/8 | | | | | | | | | | |
| 1 | 1.3736 | | | | | | | | | |
| 2 | 5.9892 | 0.0567 | | | | | | | | |
| 3 | 8.9466 | 0.1403 | 4.3787 | | | | | | | |
| 4 | 10.0512 | 0.1866 | 9.8722 | 0.0733 | | | | | | |
| 5 | 11.1128 | 0.2008 | 12.7123 | 0.1602 | 4.8368 | | | | | |
| 6 | 11.2235 | 0.2179 | 13.5071 | 0.2041 | 10.4600 | 0.0770 | | | | |
| 7 | 11.8455 | 0.2179 | 14.5445 | 0.2153 | 13.2359 | 0.1652 | 4.9726 | | | |
| 8 | 11.6822 | 0.2287 | 14.4866 | 0.2312 | 13.9349 | 0.2087 | 10.6567 | 0.0783 | | |
| 9 | 12.1681 | 0.2248 | 15.1596 | 0.2297 | 14.9251 | 0.2191 | 13.4207 | 0.1672 | 5.0298 | |
| 10 | 11.9040 | 0.2336 | 14.8765 | 0.2401 | 14.8149 | 0.2346 | 14.0087 | 0.2106 | 10.7462 | 0.0790 |
| c) r = 1/4 | | | | | | | | | | |
| 1 | 0.7631 | | | | | | | | | |
| 2 | 3.0912 | 0.1220 | | | | | | | | |
| 3 | 4.5446 | 0.2886 | 2.3272 | | | | | | | |
| 4 | 5.0046 | 0.3815 | 5.0696 | 0.1559 | | | | | | |
| 5 | 5.5547 | 0.4037 | 6.4880 | 0.3281 | 2.5577 | | | | | |
| 6 | 5.5377 | 0.4403 | 6.7916 | 0.4167 | 5.3580 | 0.1632 | | | | |
| 7 | 5.8904 | 0.4353 | 7.3489 | 0.4330 | 6.7472 | 0.3379 | 2.6256 | | | |
| 8 | 5.7441 | 0.4601 | 7.2417 | 0.4675 | 7.0032 | 0.4258 | 5.4537 | 0.1659 | | |
| 9 | 6.0374 | 0.4478 | 7.6338 | 0.4594 | 7.5404 | 0.4404 | 6.8374 | 0.3419 | 2.6542 | |
| 10 | 5.8435 | 0.4690 | 7.4193 | 0.4839 | 7.4072 | 0.4742 | 7.0778 | 0.4295 | 5.4970 | 0.1672 |
| d) r = 1/3 | | | | | | | | | | |
| 1 | 0.6105 | | | | | | | | | |
| 2 | 2.3497 | 0.1712 | | | | | | | | |
| 3 | 3.4253 | 0.3914 | 1.8216 | | | | | | | |
| 4 | 3.7120 | 0.5161 | 3.8671 | 0.2172 | | | | | | |
| 5 | 4.1422 | 0.5389 | 4.9341 | 0.4442 | 1.9966 | | | | | |
| 6 | 4.0823 | 0.5906 | 5.1046 | 0.5639 | 4.0826 | 0.2272 | | | | |
| 7 | 4.3777 | 0.5785 | 5.5512 | 0.5786 | 5.1294 | 0.4572 | 2.0481 | | | |
| 8 | 4.2247 | 0.6155 | 5.4212 | 0.6284 | 5.2644 | 0.5760 | 4.1536 | 0.2308 | | |
| 9 | 4.4804 | 0.5942 | 5.7535 | 0.6119 | 5.6978 | 0.5885 | 5.1967 | 0.4623 | 2.0697 | |
| 10 | 4.2930 | 0.6266 | 5.5455 | 0.6490 | 5.5485 | 0.6374 | 5.3199 | 0.5810 | 4.1857 | 0.2325 |
| e) r = 1/2 | | | | | | | | | | |
| 1 | 0.4579 | | | | | | | | | |
| 2 | 1.5715 | 0.2880 | | | | | | | | |
| 3 | 2.2746 | 0.6035 | 1.3341 | | | | | | | |
| 4 | 2.3545 | 0.7973 | 2.6600 | 0.3626 | | | | | | |
| 5 | 2.6921 | 0.8042 | 3.3882 | 0.6853 | 1.4572 | | | | | |
| 6 | 2.5561 | 0.8962 | 3.3962 | 0.8761 | 2.8071 | 0.3785 | | | | |
| 7 | 2.8260 | 0.8560 | 3.7594 | 0.8685 | 3.5246 | 0.7050 | 1.4932 | | | |
| 8 | 2.6324 | 0.9285 | 3.5762 | 0.9619 | 3.5095 | 0.8950 | 2.8547 | 0.3843 | | |
| 9 | 2.8839 | 0.8762 | 3.8788 | 0.9121 | 3.8660 | 0.8836 | 3.5703 | 0.7127 | 1.5084 | |
| 10 | 2.6688 | 0.9429 | 3.6461 | 0.9887 | 3.6707 | 0.9765 | 3.5472 | 0.9027 | 2.8761 | 0.3870 |
| f) r = 1 | | | | | | | | | | |
| 1 | 0.3052 | | | | | | | | | |
| 2 | - - - | - - - | | | | | | | | |
| 3 | 1.0316 | 1.1474 | 1.0316 | | | | | | | |
| 4 | - - - | - - - | - - - | - - - | | | | | | |
| 5 | 1.1468 | 1.3712 | 1.9750 | 1.3712 | 1.1468 | | | | | |
| 6 | - - - | - - - | - - - | - - - | - - - | - - - | | | | |
| 7 | 1.1812 | 1.4228 | 2.0967 | 1.5734 | 2.0967 | 1.4228 | 1.1812 | | | |
| 8 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | | |
| 9 | 1.1957 | 1.4426 | 2.1346 | 1.6167 | 2.2054 | 1.6167 | 2.1346 | 1.4426 | 1.1957 | |
| 10 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - |

Here the parameter $\epsilon$ is the ripple factor and $T_n(\omega)$ is the Tschebyscheff polynomial of order (and degree) $n$; $T_n(\omega)$ is defined by $\cos(n \cos^{-1} \omega)$. The role played by $\epsilon$ and the equal-ripple quality of the Tschebyscheff approximation are illustrated in Fig. 7, where $n = 3$ and a 1-db ripple are used.

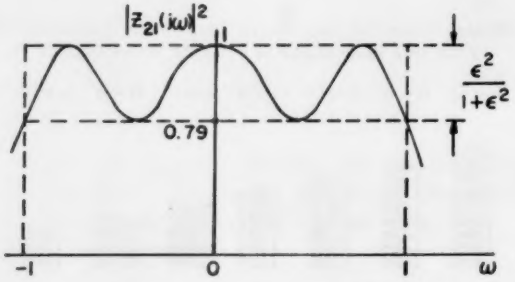


FIG. 7. Low-pass filter obtained by using the Tschebyscheff approximation with $n = 3$ and a 1-db ripple.

From the magnitude given by Eq. 3 the complete transfer function can be found. It is given by

$$Z_{21}(s) = \frac{H}{V_n(s)}, \tag{4}$$

where $H$ again is a constant and $V_n$ is formed from the left half-plane zeros of the denominator of Eq. 3; $V_n$ is a polynomial of degree $n$ with the coefficient of $s^n$ equal to unity and with its zeros lying on an ellipse.

The element values of the ladder networks for values of $\epsilon$ corresponding to $\frac{1}{10}$-, $\frac{1}{4}$-, $\frac{1}{2}$-, 1-, 2-, and 3-db ripple are presented in Tables II through VII.

After the value of $\epsilon$ has been calculated from a specified ripple factor, it is necessary to determine the required value of $n$. Formulas useful for this purpose are

$$T_n(\omega) = \frac{(\omega + \sqrt{\omega^2 - 1})^n + (\omega + \sqrt{\omega^2 - 1})^{-n}}{2} \tag{5}$$

or

$$T_n(\omega) = \frac{(\omega - \sqrt{\omega^2 - 1})^n + (\omega - \sqrt{\omega^2 - 1})^{-n}}{2}. \tag{6}$$

TABLE III.—*Element Values (in ohms, henrys, farads) for a Normalized Tschebyscheff Filter with ½-db Ripple ($\epsilon = 0.2434$, $\epsilon^2 = 0.0593$).*

| Value of n | $C_1$ or $L_1'$ | $L_2$ or $C_2'$ | $C_3$ or $L_3'$ | $L_4$ or $C_4'$ | $C_5$ or $L_5'$ | $L_6$ or $C_6'$ | $C_7$ or $L_7'$ | $L_8$ or $C_8'$ | $C_9$ or $L_9'$ | $L_{10}$ or $C_{10}'$ |
|---|---|---|---|---|---|---|---|---|---|---|
| a) r = 0 | | | | | | | | | | |
| 1 | 0.2434 | | | | | | | | | |
| 2 | 0.5566 | 0.8499 | | | | | | | | |
| 3 | 0.6517 | 1.2198 | 1.2248 | | | | | | | |
| 4 | 0.6891 | 1.3215 | 1.5979 | 1.3003 | | | | | | |
| 5 | 0.6912 | 1.3538 | 1.6741 | 1.6371 | 1.4480 | | | | | |
| 6 | 0.7173 | 1.3868 | 1.7271 | 1.7144 | 1.8105 | 1.4193 | | | | |
| 7 | 0.7234 | 1.3999 | 1.7475 | 1.7450 | 1.8816 | 1.7497 | 1.5323 | | | |
| 8 | 0.7274 | 1.4083 | 1.7598 | 1.7612 | 1.9099 | 1.8124 | 1.8806 | 1.4647 | | |
| 9 | 0.7302 | 1.4140 | 1.7678 | 1.7711 | 1.9248 | 1.8365 | 1.9439 | 1.7927 | 1.5648 | |
| 10 | 0.7322 | 1.4180 | 1.7733 | 1.7776 | 1.9338 | 1.8490 | 1.9676 | 1.8505 | 1.9119 | 1.4864 |
| b) r = 1/8 | | | | | | | | | | |
| 1 | 2.1908 | | | | | | | | | |
| 2 | 7.0446 | 0.0755 | | | | | | | | |
| 3 | 10.0648 | 0.1562 | 5.5746 | | | | | | | |
| 4 | 10.4126 | 0.2051 | 10.8046 | 0.0922 | | | | | | |
| 5 | 11.7227 | 0.2052 | 13.6987 | 0.1726 | 5.8758 | | | | | |
| 6 | 11.2734 | 0.2298 | 13.7315 | 0.2211 | 11.3063 | 0.0958 | | | | |
| 7 | 12.3557 | 0.2179 | 15.2643 | 0.2183 | 14.2825 | 0.1780 | 6.1472 | | | |
| 8 | 11.5987 | 0.2378 | 14.4405 | 0.2423 | 14.0925 | 0.2250 | 11.4700 | 0.0970 | | |
| 9 | 12.5942 | 0.2228 | 15.7237 | 0.2287 | 15.6054 | 0.2214 | 14.4391 | 0.1797 | 6.2011 | |
| 10 | 11.7539 | 0.2414 | 14.7148 | 0.2489 | 14.7268 | 0.2452 | 14.2166 | 0.2267 | 11.5438 | 0.0976 |
| c) r = 1/4 | | | | | | | | | | |
| 1 | 1.2171 | | | | | | | | | |
| 2 | 3.5907 | 0.1647 | | | | | | | | |
| 3 | 5.1234 | 0.3182 | 2.9867 | | | | | | | |
| 4 | 5.1282 | 0.4214 | 5.4989 | 0.1990 | | | | | | |
| 5 | 5.8822 | 0.4097 | 7.0092 | 0.3509 | 3.1328 | | | | | |
| 6 | 5.5080 | 0.4669 | 6.8423 | 0.4540 | 5.7437 | 0.2062 | | | | |
| 7 | 6.1786 | 0.4322 | 7.7470 | 0.4354 | 7.3060 | 0.3611 | 3.2774 | | | |
| 8 | 5.6503 | 0.4815 | 7.1583 | 0.4930 | 7.0209 | 0.4618 | 5.8228 | 0.2087 | | |
| 9 | 6.2867 | 0.4410 | 7.9582 | 0.4543 | 7.9206 | 0.4415 | 7.3830 | 0.3643 | 3.3046 | |
| 10 | 5.7179 | 0.4880 | 7.2798 | 0.5050 | 7.3032 | 0.4989 | 7.0810 | 0.4650 | 5.8583 | 0.2099 |
| d) r = 1/3 | | | | | | | | | | |
| 1 | 0.9737 | | | | | | | | | |
| 2 | 2.6983 | 0.2337 | | | | | | | | |
| 3 | 3.8716 | 0.4279 | 2.3508 | | | | | | | |
| 4 | 3.7640 | 0.5729 | 4.1610 | 0.2812 | | | | | | |
| 5 | 4.4034 | 0.5439 | 5.3427 | 0.4720 | 2.4596 | | | | | |
| 6 | 4.0210 | 0.6299 | 5.1006 | 0.6179 | 4.3438 | 0.2910 | | | | |
| 7 | 4.6162 | 0.5713 | 5.8745 | 0.5783 | 5.5708 | 0.4851 | 2.5735 | | | |
| 8 | 4.1167 | 0.6480 | 5.3173 | 0.6668 | 5.2361 | 0.6285 | 4.4023 | 0.2945 | | |
| 9 | 4.6917 | 0.5822 | 6.0240 | 0.6017 | 6.0086 | 0.5864 | 5.6288 | 0.4893 | 2.5944 | |
| 10 | 4.1620 | 0.6561 | 5.4002 | 0.6817 | 5.4294 | 0.6749 | 5.2808 | 0.6328 | 4.4285 | 0.2962 |
| e) r = 1/2 | | | | | | | | | | |
| 1 | 0.7303 | | | | | | | | | |
| 2 | 1.7288 | 0.4104 | | | | | | | | |
| 3 | 2.5965 | 0.6465 | 1.7402 | | | | | | | |
| 4 | 2.2884 | 0.9039 | 2.7832 | 0.4930 | | | | | | |
| 5 | 2.8983 | 0.8007 | 3.6926 | 0.7162 | 1.8152 | | | | | |
| 6 | 2.4162 | 0.9771 | 3.2941 | 0.9837 | 2.9094 | 0.5100 | | | | |
| 7 | 3.0294 | 0.8341 | 4.0204 | 0.8546 | 3.8585 | 0.7340 | 1.9007 | | | |
| 8 | 2.4631 | 1.0000 | 3.4072 | 1.0463 | 3.3925 | 1.0015 | 2.9490 | 0.5161 | | |
| 9 | 3.0724 | 0.8478 | 4.1088 | 0.8843 | 4.1199 | 0.8670 | 3.8989 | 0.7404 | 1.9157 | |
| 10 | 2.4852 | 1.0100 | 3.4501 | 1.0651 | 3.4927 | 1.0606 | 3.4235 | 1.0085 | 2.9666 | 0.5190 |
| f) r = 1 | | | | | | | | | | |
| 1 | 0.4868 | | | | | | | | | |
| 2 | - - - | - - - | | | | | | | | |
| 3 | 1.3034 | 1.1463 | 1.3034 | | | | | | | |
| 4 | - - - | - - - | - - - | - - - | | | | | | |
| 5 | 1.3824 | 1.3264 | 2.2091 | 1.3264 | 1.3824 | | | | | |
| 6 | - - - | - - - | - - - | - - - | - - - | - - - | | | | |
| 7 | 1.4468 | 1.3560 | 2.3476 | 1.4689 | 2.3476 | 1.3560 | 1.4468 | | | |
| 8 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | | |
| 9 | 1.4604 | 1.3704 | 2.3800 | 1.5000 | 2.4414 | 1.5000 | 2.3800 | 1.3704 | 1.4604 | |
| 10 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - |

TABLE IV.—*Element Values (in ohms, henrys, farads) for a Normalized Tschebyscheff Filter (½-db Ripple ($\epsilon = 0.3493$, $\epsilon^2 = 0.1220$).*

| Value of n | $C_1$ or $L_1'$ | $L_2$ or $C_2'$ | $C_3$ or $L_3'$ | $L_4$ or $C_4'$ | $C_5$ or $L_5'$ | $L_6$ or $C_6'$ | $C_7$ or $L_7'$ | $L_8$ or $C_8'$ | $C_9$ or $L_9'$ | $L_{10}$ or $C_{10}'$ |
|---|---|---|---|---|---|---|---|---|---|---|
| a) r = 0 | | | | | | | | | | |
| 1 | 0.3493 | | | | | | | | | |
| 2 | 0.7014 | 0.9403 | | | | | | | | |
| 3 | 0.7981 | 1.3001 | 1.3465 | | | | | | | |
| 4 | 0.8352 | 1.3916 | 1.7279 | 1.3138 | | | | | | |
| 5 | 0.8529 | 1.4291 | 1.8142 | 1.6426 | 1.5388 | | | | | |
| 6 | 0.8627 | 1.4483 | 1.8494 | 1.7101 | 1.9018 | 1.4042 | | | | |
| 7 | 0.8686 | 1.4596 | 1.8675 | 1.7371 | 1.9712 | 1.7254 | 1.5982 | | | |
| 8 | 0.8725 | 1.4666 | 1.8750 | 1.7508 | 1.9980 | 1.7838 | 1.9571 | 1.4379 | | |
| 9 | 0.8752 | 1.4714 | 1.8856 | 1.7591 | 2.0116 | 1.8055 | 2.0203 | 1.7571 | 1.6238 | |
| 10 | 0.8771 | 1.4748 | 1.8905 | 1.7645 | 2.0197 | 1.8165 | 2.0432 | 1.8119 | 1.9816 | 1.4539 |
| b) r = 1/8 | | | | | | | | | | |
| 1 | 3.1438 | | | | | | | | | |
| 2 | 7.6905 | 0.0965 | | | | | | | | |
| 3 | 11.1053 | 0.1646 | 6.8796 | | | | | | | |
| 4 | 10.3991 | 0.2234 | 11.2532 | 0.1135 | | | | | | |
| 5 | 12.5367 | 0.2039 | 14.9223 | 0.1801 | 7.3211 | | | | | |
| 6 | 11.0346 | 0.2434 | 13.5532 | 0.2386 | 11.6839 | 0.1170 | | | | |
| 7 | 12.9745 | 0.2131 | 16.0900 | 0.2154 | 15.3437 | 0.1837 | 7.4478 | | | |
| 8 | 11.2694 | 0.2497 | 14.0704 | 0.2556 | 13.8641 | 0.2422 | 11.8216 | 0.1182 | | |
| 9 | 13.1608 | 0.2167 | 16.4502 | 0.2230 | 16.4112 | 0.2180 | 15.4833 | 0.1852 | 7.5006 | |
| 10 | 11.3804 | 0.2524 | 14.2670 | 0.2607 | 14.3247 | 0.2583 | 13.9570 | 0.2436 | 11.8832 | 0.1188 |
| c) r = 1/4 | | | | | | | | | | |
| 1 | 1.7466 | | | | | | | | | |
| 2 | 3.8432 | 0.2145 | | | | | | | | |
| 3 | 5.6859 | 0.3308 | 3.7139 | | | | | | | |
| 4 | 5.0293 | 0.4646 | 5.6377 | 0.2504 | | | | | | |
| 5 | 6.3476 | 0.4023 | 7.6862 | 0.3613 | 3.9411 | | | | | |
| 6 | 5.2985 | 0.5009 | 6.6492 | 0.4963 | 5.8461 | 0.2576 | | | | |
| 7 | 6.5476 | 0.4187 | 8.2283 | 0.4252 | 7.8993 | 0.3683 | 4.0061 | | | |
| 8 | 5.3972 | 0.5122 | 6.8706 | 0.5272 | 6.8027 | 0.5035 | 5.9120 | 0.2602 | | |
| 9 | 6.6323 | 0.4250 | 8.3941 | 0.4389 | 8.3940 | 0.4303 | 7.9689 | 0.3710 | 4.0331 | |
| 10 | 5.4437 | 0.5172 | 6.9544 | 0.5364 | 6.9993 | 0.5328 | 6.8522 | 0.5064 | 5.9413 | 0.2614 |
| d) r = 1/3 | | | | | | | | | | |
| 1 | 1.3972 | | | | | | | | | |
| 2 | 2.8282 | 0.3109 | | | | | | | | |
| 3 | 4.3200 | 0.4405 | 2.9371 | | | | | | | |
| 4 | 3.6172 | 0.6399 | 4.1985 | 0.3620 | | | | | | |
| 5 | 4.7896 | 0.5293 | 5.8898 | 0.4809 | 3.1130 | | | | | |
| 6 | 3.7922 | 0.6851 | 4.8770 | 0.6852 | 4.3536 | 0.3722 | | | | |
| 7 | 4.9305 | 0.5495 | 6.2770 | 0.5603 | 6.0535 | 0.4901 | 3.1632 | | | |
| 8 | 3.8560 | 0.6990 | 5.0230 | 0.7235 | 4.9937 | 0.6953 | 4.4022 | 0.3759 | | |
| 9 | 4.9901 | 0.5572 | 6.3947 | 0.5770 | 6.4061 | 0.5671 | 6.1064 | 0.4936 | 3.1841 | |
| 10 | 3.8860 | 0.7051 | 5.0780 | 0.7348 | 5.1229 | 0.7314 | 5.0307 | 0.6993 | 4.4237 | 0.3776 |
| e) r = 1/2 | | | | | | | | | | |
| 1 | 1.0479 | | | | | | | | | |
| 2 | 1.5132 | 0.6538 | | | | | | | | |
| 3 | 2.9431 | 0.6503 | 2.1903 | | | | | | | |
| 4 | 1.8158 | 1.1328 | 2.4881 | 0.7732 | | | | | | |
| 5 | 3.2228 | 0.7645 | 4.1228 | 0.7116 | 2.3197 | | | | | |
| 6 | 1.8786 | 1.1884 | 2.7589 | 1.2403 | 2.5976 | 0.7976 | | | | |
| 7 | 3.3055 | 0.7899 | 4.3575 | 0.8132 | 4.2419 | 0.7252 | 2.3566 | | | |
| 8 | 1.9012 | 1.2053 | 2.8152 | 1.2864 | 2.8479 | 1.2628 | 2.6310 | 0.8063 | | |
| 9 | 3.3403 | 0.7995 | 4.4283 | 0.8341 | 4.4546 | 0.8235 | 4.2795 | 0.7304 | 2.3719 | |
| 10 | 1.9117 | 1.2127 | 2.8366 | 1.2999 | 2.8964 | 1.3054 | 2.8744 | 1.2714 | 2.6456 | 0.8104 |
| f) r = 1 | | | | | | | | | | |
| 1 | 0.6986 | | | | | | | | | |
| 2 | - - - | - - - | | | | | | | | |
| 3 | 1.5963 | 1.0967 | 1.5963 | | | | | | | |
| 4 | - - - | - - - | - - - | - - - | | | | | | |
| 5 | 1.7058 | 1.2296 | 2.5408 | 1.2296 | 1.7058 | | | | | |
| 6 | - - - | - - - | - - - | - - - | - - - | - - - | | | | |
| 7 | 1.7373 | 1.2582 | 2.6383 | 1.3443 | 2.6383 | 1.2582 | 1.7373 | | | |
| 8 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | | |
| 9 | 1.7504 | 1.2690 | 2.6678 | 1.3673 | 2.7239 | 1.3673 | 2.6678 | 1.2690 | 1.7504 | |
| 10 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - |

TABLE V.—*Element Values (in ohms, henrys, farads) for a Normalized Tschebyscheff Filter with 1-db Ripple* ($\epsilon = 0.5088$, $\epsilon^2 = 0.2589$).

| Value of n | $C_1$ or $L_1'$ | $L_2$ or $C_2'$ | $C_3$ or $L_3'$ | $L_4$ or $C_4'$ | $C_5$ or $L_5'$ | $L_6$ or $C_6'$ | $C_7$ or $L_7'$ | $L_8$ or $C_8'$ | $C_9$ or $L_9'$ | $L_{10}$ or $C_{10}'$ |
|---|---|---|---|---|---|---|---|---|---|---|
| a) r = 0 | | | | | | | | | | |
| 1 | 0.5088 | | | | | | | | | |
| 2 | 0.9110 | 0.9957 | | | | | | | | |
| 3 | 1.0118 | 1.3332 | 1.5088 | | | | | | | |
| 4 | 1.0495 | 1.4126 | 1.9093 | 1.2817 | | | | | | |
| 5 | 1.0674 | 1.4441 | 1.9938 | 1.5908 | 1.6652 | | | | | |
| 6 | 1.0773 | 1.4601 | 2.0270 | 1.6507 | 2.0491 | 1.3457 | | | | |
| 7 | 1.0832 | 1.4694 | 2.0437 | 1.6736 | 2.1192 | 1.6489 | 1.7118 | | | |
| 8 | 1.0872 | 1.4751 | 2.0537 | 1.6850 | 2.1453 | 1.7021 | 2.0922 | 1.3691 | | |
| 9 | 1.0899 | 1.4790 | 2.0601 | 1.6918 | 2.1583 | 1.7213 | 2.1574 | 1.6707 | 1.7317 | |
| 10 | 1.0918 | 1.4817 | 2.0645 | 1.6961 | 2.1658 | 1.7306 | 2.1803 | 1.7215 | 2.1111 | 1.3801 |
| b) r = 1/8 | | | | | | | | | | |
| 1 | 4.5796 | | | | | | | | | |
| 2 | 7.9318 | 0.1286 | | | | | | | | |
| 3 | 12.5563 | 0.1657 | 8.8038 | | | | | | | |
| 4 | 9.9024 | 0.2517 | 11.1584 | 0.1467 | | | | | | |
| 5 | 13.7259 | 0.1945 | 16.5650 | 0.1789 | 9.2596 | | | | | |
| 6 | 10.3304 | 0.2677 | 12.7878 | 0.2668 | 11.5115 | 0.1503 | | | | |
| 7 | 14.0719 | 0.2009 | 17.5013 | 0.2045 | 16.9660 | 0.1819 | 9.3890 | | | |
| 8 | 10.4856 | 0.2725 | 13.1313 | 0.2802 | 13.0465 | 0.2701 | 11.6220 | 0.1516 | | |
| 9 | 14.2174 | 0.2033 | 17.7827 | 0.2097 | 17.8168 | 0.2066 | 17.0949 | 0.1830 | 9.4427 | |
| 10 | 10.5585 | 0.2746 | 13.2602 | 0.2842 | 13.3503 | 0.2828 | 13.1287 | 0.2715 | 11.6709 | 0.1522 |
| c) r = 1/4 | | | | | | | | | | |
| 1 | 2.5442 | | | | | | | | | |
| 2 | 3.7779 | 0.3001 | | | | | | | | |
| 3 | 6.5048 | 0.3264 | 4.7927 | | | | | | | |
| 4 | 4.5699 | 0.5428 | 5.3680 | 0.3406 | | | | | | |
| 5 | 7.0522 | 0.3776 | 8.6301 | 0.3520 | 5.0313 | | | | | |
| 6 | 4.7366 | 0.5716 | 6.0240 | 0.5764 | 5.5353 | 0.3486 | | | | |
| 7 | 7.2126 | 0.3888 | 9.0689 | 0.3973 | 8.8368 | 0.3577 | 5.0989 | | | |
| 8 | 4.7966 | 0.5803 | 6.1592 | 0.6005 | 6.1501 | 0.5836 | 5.5869 | 0.3515 | | |
| 9 | 7.2800 | 0.3930 | 9.2001 | 0.4064 | 9.2344 | 0.4015 | 8.9024 | 0.3598 | 5.1270 | |
| 10 | 4.8247 | 0.5841 | 6.2098 | 0.6076 | 6.2689 | 0.6063 | 6.1890 | 0.5864 | 5.6096 | 0.3528 |
| d) r = 1/3 | | | | | | | | | | |
| 1 | 2.0354 | | | | | | | | | |
| 2 | 2.5721 | 0.4702 | | | | | | | | |
| 3 | 4.9893 | 0.4286 | 3.8075 | | | | | | | |
| 4 | 3.0355 | 0.7929 | 3.7589 | 0.5347 | | | | | | |
| 5 | 5.3830 | 0.4915 | 6.6673 | 0.4622 | 3.9944 | | | | | |
| 6 | 3.1307 | 0.8287 | 4.1451 | 0.8467 | 3.8812 | 0.5475 | | | | |
| 7 | 5.4978 | 0.5050 | 6.9839 | 0.5177 | 6.8280 | 0.4696 | 4.0473 | | | |
| 8 | 3.1647 | 0.8395 | 4.2237 | 0.8764 | 4.2404 | 0.8580 | 3.9186 | 0.5520 | | |
| 9 | 5.5459 | 0.5101 | 7.0783 | 0.5288 | 7.1141 | 0.5232 | 6.8785 | 0.4724 | 4.0693 | |
| 10 | 3.1806 | 0.8442 | 4.2532 | 0.8851 | 4.3088 | 0.8857 | 4.2691 | 0.8623 | 3.9349 | 0.5541 |
| e) r = 1/2 | | | | | | | | | | |
| 1 | 1.5265 | | | | | | | | | |
| 2 | - - - | - - - | | | | | | | | |
| 3 | 3.4774 | 0.6153 | 2.8540 | | | | | | | |
| 4 | - - - | - - - | - - - | - - - | | | | | | |
| 5 | 3.7211 | 0.6949 | 4.7448 | 0.6650 | 2.9936 | | | | | |
| 6 | - - - | - - - | - - - | - - - | - - - | - - - | | | | |
| 7 | 3.7916 | 0.7118 | 4.9425 | 0.7348 | 4.8636 | 0.6757 | 3.0331 | | | |
| 8 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | | |
| 9 | 3.8210 | 0.7182 | 5.0013 | 0.7485 | 5.0412 | 0.7429 | 4.9004 | 0.6797 | 3.0495 | |
| 10 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - |
| f) r = 1 | | | | | | | | | | |
| 1 | 1.0177 | | | | | | | | | |
| 2 | - - - | - - - | | | | | | | | |
| 3 | 2.0236 | 0.9941 | 2.0236 | | | | | | | |
| 4 | - - - | - - - | - - - | - - - | | | | | | |
| 5 | 2.1349 | 1.0911 | 3.0009 | 1.0911 | 2.1349 | | | | | |
| 6 | - - - | - - - | - - - | - - - | - - - | - - - | | | | |
| 7 | 2.1666 | 1.1115 | 3.0936 | 1.1735 | 3.0936 | 1.1115 | 2.1666 | | | |
| 8 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | | |
| 9 | 2.1797 | 1.1192 | 3.1214 | 1.1897 | 3.1746 | 1.1897 | 3.1214 | 1.1192 | 2.1797 | |
| 10 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - |

TABLE VI.—*Element Values (in ohms, henrys, farads) for a Normalized Tschebyscheff Filter with 2-db Ripple* ($\epsilon = 0.7648$, $\epsilon^2 = 0.5849$).

| Value of n | $C_1$ or $L_1'$ | $L_2$ or $C_2'$ | $C_3$ or $L_3'$ | $L_4$ or $C_4'$ | $C_5$ or $L_5'$ | $L_6$ or $C_6'$ | $C_7$ or $L_7'$ | $L_8$ or $C_8'$ | $C_9$ or $L_9'$ | $L_{10}$ or $C_{10}'$ |
|---|---|---|---|---|---|---|---|---|---|---|
| a) r = 0 | | | | | | | | | | |
| 1 | 0.7648 | | | | | | | | | |
| 2 | 1.2441 | 0.9766 | | | | | | | | |
| 3 | 1.3553 | 1.2740 | 1.7717 | | | | | | | |
| 4 | 1.3962 | 1.3389 | 2.2169 | 1.1727 | | | | | | |
| 5 | 1.4155 | 1.3640 | 2.3049 | 1.4668 | 1.9004 | | | | | |
| 6 | 1.4261 | 1.3765 | 2.3383 | 1.4974 | 2.3304 | 1.2137 | | | | |
| 7 | 1.4328 | 1.3836 | 2.3551 | 1.5159 | 2.4063 | 1.4836 | 1.9379 | | | |
| 8 | 1.4366 | 1.3881 | 2.3645 | 1.5251 | 2.4332 | 1.5298 | 2.3646 | 1.2284 | | |
| 9 | 1.4395 | 1.3911 | 2.3707 | 1.5304 | 2.4463 | 1.5495 | 2.4386 | 1.4959 | 1.9553 | |
| 10 | 1.4416 | 1.3932 | 2.3748 | 1.5337 | 2.4538 | 1.5536 | 2.4607 | 1.5419 | 2.3794 | 1.2353 |
| b) r = 1/8 | | | | | | | | | | |
| 1 | 6.8830 | | | | | | | | | |
| 2 | 7.2895 | 0.1875 | | | | | | | | |
| 3 | 14.9900 | 0.1541 | 11.9205 | | | | | | | |
| 4 | 8.5051 | 0.3093 | 9.9546 | 0.2088 | | | | | | |
| 5 | 15.9745 | 0.1729 | 19.4874 | 0.1646 | 12.4250 | | | | | |
| 6 | 8.7527 | 0.3224 | 10.9256 | 0.3260 | 10.2196 | 0.2130 | | | | |
| 7 | 16.2581 | 0.1769 | 20.2574 | 0.1811 | 19.8989 | 0.1669 | 12.5671 | | | |
| 8 | 8.8412 | 0.3263 | 11.1205 | 0.3368 | 11.1251 | 0.3295 | 10.3007 | 0.2144 | | |
| 9 | 16.3765 | 0.1784 | 20.4848 | 0.1844 | 20.5920 | 0.1828 | 20.0271 | 0.1678 | 12.6259 | |
| 10 | 8.8824 | 0.3280 | 11.1932 | 0.3399 | 11.2954 | 0.3396 | 11.1858 | 0.3308 | 10.3363 | 0.2151 |
| c) r = 1/4 | | | | | | | | | | |
| 1 | 3.8239 | | | | | | | | | |
| 2 | - - - | - - - | | | | | | | | |
| 3 | 7.9106 | 0.2955 | 6.5423 | | | | | | | |
| 4 | - - - | - - - | - - - | - - - | | | | | | |
| 5 | 8.3859 | 0.3285 | 10.3300 | 0.3156 | 6.8118 | | | | | |
| 6 | - - - | - - - | - - - | - - - | - - - | - - - | | | | |
| 7 | 8.5220 | 0.3354 | 10.7009 | 0.3443 | 10.5472 | 0.3199 | 6.8877 | | | |
| 8 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | | |
| 9 | 8.5787 | 0.3380 | 10.8103 | 0.3499 | 10.8798 | 0.3475 | 10.6142 | 0.3215 | 6.9191 | |
| 10 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - |
| d) r = 1/3 | | | | | | | | | | |
| 1 | 3.0591 | | | | | | | | | |
| 2 | - - - | - - - | | | | | | | | |
| 3 | 6.1471 | 0.3816 | 5.2161 | | | | | | | |
| 4 | - - - | - - - | - - - | - - - | | | | | | |
| 5 | 6.4974 | 0.4219 | 8.0681 | 0.4076 | 5.4294 | | | | | |
| 6 | - - - | - - - | - - - | - - - | - - - | - - - | | | | |
| 7 | 6.5974 | 0.4302 | 8.3415 | 0.4425 | 8.2389 | 0.4131 | 5.4893 | | | |
| 8 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | | |
| 9 | 6.6391 | 0.4334 | 8.4220 | 0.4493 | 8.4834 | 0.4467 | 8.2913 | 0.4152 | 5.5141 | |
| 10 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - |
| e) r = 1/2 | | | | | | | | | | |
| 1 | 2.2943 | | | | | | | | | |
| 2 | - - - | - - - | | | | | | | | |
| 3 | 4.3975 | 0.5326 | 3.9184 | | | | | | | |
| 4 | - - - | - - - | - - - | - - - | | | | | | |
| 5 | 4.6265 | 0.5835 | 5.8503 | 0.5698 | 4.0790 | | | | | |
| 6 | - - - | - - - | - - - | - - - | - - - | - - - | | | | |
| 7 | 4.6917 | 0.5941 | 6.0293 | 0.6136 | 5.9780 | 0.5776 | 4.1242 | | | |
| 8 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | | |
| 9 | 4.7187 | 0.5980 | 6.0821 | 0.6220 | 6.1370 | 0.6195 | 6.0168 | 0.5805 | 4.1429 | |
| 10 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - |
| f) r = 1 | | | | | | | | | | |
| 1 | 1.5296 | | | | | | | | | |
| 2 | - - - | - - - | | | | | | | | |
| 3 | 2.7107 | 0.8327 | 2.7107 | | | | | | | |
| 4 | - - - | - - - | - - - | - - - | | | | | | |
| 5 | 2.8310 | 0.8985 | 3.7827 | 0.8985 | 2.8310 | | | | | |
| 6 | - - - | - - - | - - - | - - - | - - - | - - - | | | | |
| 7 | 2.8650 | 0.9120 | 3.8774 | 0.9537 | 3.8774 | 0.9120 | 2.8650 | | | |
| 8 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | | |
| 9 | 2.8790 | 0.9171 | 3.9056 | 0.9643 | 3.9597 | 0.9643 | 3.9056 | 0.9171 | 2.8790 | |
| 10 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - |

TABLE VII.—*Element Values (in ohms, henrys, farads) for a Normalized Tschebyscheff Filter with 3-db Ripple ($\epsilon = 0.9976$, $\epsilon^2 = 0.9953$).*

| Value of n | $C_1$ or $L_1'$ | $L_2$ or $C_2'$ | $C_3$ or $L_3'$ | $L_4$ or $C_4'$ | $C_5$ or $L_5'$ | $L_6$ or $C_6'$ | $C_7$ or $L_7'$ | $L_8$ or $C_8'$ | $C_9$ or $L_9'$ | $L_{10}$ or $C_{10}'$ |
|---|---|---|---|---|---|---|---|---|---|---|
| a) r = 0 | | | | | | | | | | |
| 1 | 0.9976 | | | | | | | | | |
| 2 | 1.5506 | 0.9109 | | | | | | | | |
| 3 | 1.6744 | 1.1739 | 2.0302 | | | | | | | |
| 4 | 1.7195 | 1.2292 | 2.5272 | 1.0578 | | | | | | |
| 5 | 1.7409 | 1.2501 | 2.6227 | 1.3015 | 2.4491 | | | | | |
| 6 | 1.7522 | 1.2606 | 2.6578 | 1.3455 | 2.6309 | 1.0876 | | | | |
| 7 | 1.7591 | 1.2666 | 2.6750 | 1.3614 | 2.7141 | 1.3282 | 2.1827 | | | |
| 8 | 1.7638 | 1.2701 | 2.6852 | 1.3690 | 2.7436 | 1.3687 | 2.6618 | 1.0982 | | |
| 9 | 1.7670 | 1.2726 | 2.6916 | 1.3733 | 2.7577 | 1.3827 | 2.7414 | 1.3380 | 2.1970 | |
| 10 | 1.7692 | 1.2744 | 2.6958 | 1.3761 | 2.7655 | 1.3893 | 2.7683 | 1.3774 | 2.6753 | 1.1032 |
| b) r = 1/8 | | | | | | | | | | |
| 1 | 8.9787 | | | | | | | | | |
| 2 | 6.1219 | 0.2596 | | | | | | | | |
| 3 | 17.4070 | 0.1392 | 14.8205 | | | | | | | |
| 4 | 6.9104 | 0.3884 | 8.2760 | 0.2861 | | | | | | |
| 5 | 18.3377 | 0.1530 | 22.4760 | 0.1481 | 15.3856 | | | | | |
| 6 | 7.0661 | 0.4011 | 8.8887 | 0.4087 | 8.4796 | 0.2913 | | | | |
| 7 | 18.6027 | 0.1559 | 23.1920 | 0.1600 | 22.9195 | 0.1499 | 15.5441 | | | |
| 8 | 7.1213 | 0.4048 | 9.0096 | 0.4190 | 9.0452 | 0.4128 | 8.5411 | 0.2931 | | |
| 9 | 18.7129 | 0.1569 | 23.4023 | 0.1623 | 23.5592 | 0.1614 | 23.0554 | 0.1506 | 15.6097 | |
| 10 | 7.1470 | 0.4064 | 9.0546 | 0.4219 | 9.1496 | 0.4223 | 9.0917 | 0.4144 | 8.5679 | 0.2939 |
| c) r = 1/4 | | | | | | | | | | |
| 1 | 4.9881 | | | | | | | | | |
| 2 | - - - | - - - | | | | | | | | |
| 3 | 9.3059 | 0.2625 | 8.1669 | | | | | | | |
| 4 | - - - | - - - | - - - | - - - | | | | | | |
| 5 | 9.7676 | 0.2866 | 12.0571 | 0.2791 | 8.4724 | | | | | |
| 6 | - - - | - - - | - - - | - - - | - - - | - - - | | | | |
| 7 | 9.8986 | 0.2915 | 12.4111 | 0.2998 | 12.2946 | 0.2826 | 8.5581 | | | |
| 8 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | | |
| 9 | 9.9530 | 0.2934 | 12.5151 | 0.3037 | 12.6097 | 0.3024 | 12.3669 | 0.2839 | 8.5935 | |
| 10 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - |
| d) r = 1/3 | | | | | | | | | | |
| 1 | 3.9905 | | | | | | | | | |
| 2 | - - - | - - - | | | | | | | | |
| 3 | 7.2903 | 0.3358 | 6.5207 | | | | | | | |
| 4 | - - - | - - - | - - - | - - - | | | | | | |
| 5 | 7.6371 | 0.3652 | 9.4808 | 0.3571 | 6.7635 | | | | | |
| 6 | - - - | - - - | - - - | - - - | - - - | - - - | | | | |
| 7 | 7.7352 | 0.3712 | 9.7463 | 0.3822 | 9.6687 | 0.3615 | 6.8315 | | | |
| 8 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | | |
| 9 | 7.7760 | 0.3734 | 9.8243 | 0.3870 | 9.9043 | 0.3856 | 9.7256 | 0.3632 | 6.8597 | |
| 10 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - |
| e) r = 1/2 | | | | | | | | | | |
| 1 | 2.9929 | | | | | | | | | |
| 2 | - - - | - - - | | | | | | | | |
| 3 | 5.2910 | 0.4618 | 4.8991 | | | | | | | |
| 4 | - - - | - - - | - - - | - - - | | | | | | |
| 5 | 5.5259 | 0.4993 | 6.9460 | 0.4917 | 5.0821 | | | | | |
| 6 | - - - | - - - | - - - | - - - | - - - | - - - | | | | |
| 7 | 5.5922 | 0.5069 | 7.1256 | 0.5235 | 7.0869 | 0.4979 | 5.1335 | | | |
| 8 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | | |
| 9 | 5.6197 | 0.5098 | 7.1785 | 0.5296 | 7.2454 | 0.5282 | 7.1292 | 0.5002 | 5.1547 | |
| 10 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - |
| f) r = 1 | | | | | | | | | | |
| 1 | 1.9953 | | | | | | | | | |
| 2 | - - - | - - - | | | | | | | | |
| 3 | 3.3487 | 0.7117 | 3.3487 | | | | | | | |
| 4 | - - - | - - - | - - - | - - - | | | | | | |
| 5 | 3.4813 | 0.7619 | 4.5375 | 0.7619 | 3.4813 | | | | | |
| 6 | - - - | - - - | - - - | - - - | - - - | - - - | | | | |
| 7 | 3.5185 | 0.7722 | 4.6390 | 0.8038 | 4.6390 | 0.7722 | 3.5185 | | | |
| 8 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | | |
| 9 | 3.5339 | 0.7760 | 4.6691 | 0.8118 | 4.7270 | 0.8118 | 4.6691 | 0.7760 | 3.5339 | |
| 10 | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - | - - - |

The use of the tables is illustrated in the example below.

*Example 3.1.* Determine a ladder network that has the following characteristics:

1. Low-pass filter with a peak-to-peak ripple in the squared magnitude characteristic not exceeding 15 per cent of the maximum value.
2. A cutoff radian frequency $\omega_c = 5000$ (the bandwidth being measured at the minimum value of the ripple).
3. Resistance terminations at both ends with the load and input resistances equal to 1000 ohms and 500 ohms, respectively.
4. The response is to be down at least 50 db at $\omega = 4\omega_c$.
5. The network is to be driven by a current source.

We first calculate the required value of $\epsilon^2$. At a trough of the ripple we have

$$\frac{1}{1 + \epsilon^2 T_n^2(1)} = 1 - 0.15 = 0.85$$

$$1 + \epsilon^2 = \frac{20}{17}$$

$$\epsilon^2 = 0.176.$$

Since this value lies between $\frac{1}{2}$-db and 1-db ripple we must use Table IV.

Now we calculate $n$. At $\omega = 4$

$$\frac{1}{1 + \epsilon^2 T_n^2(4)} = 10^{-5}$$

$$1 + \epsilon^2 T_n^2(4) = 10^5$$

$$\epsilon^2 T_n^2(4) \cong 10^5$$

$$T_n(4) = 753.$$

Now using Eq. 5, we have

$$\left.\frac{(\omega + \sqrt{\omega^2 - 1})^n + (\omega + \sqrt{\omega^2 - 1})^{-n}}{2}\right|_{\omega=4} = 753$$

$$(\omega + \sqrt{\omega^2 - 1})^n\big|_{\omega=4} \cong 1506$$

$$(7.88)^n = 1506$$

$$n = 3.58.$$

Therefore $n = 4$ will be more than satisfactory.

Since the specification calls for $r = \frac{1}{2}$, we use Table IV($e$). Since the input is a current source the unprimed values are used. Removing the normalization by multiplying all $C$'s by $\frac{1}{R\omega_c} = \frac{1}{5 \times 10^6}$, all $L$'s by

$\frac{R}{\omega_c} = \frac{1}{5}$, and the resistances by $R = 1000$, we obtain the final element values

$$\frac{1}{5 \times 10^6} C_1 = 0.363 \times 10^{-6}$$

$$\frac{1}{5} L_2 = 0.227$$

$$\frac{1}{5 \times 10^6} C_3 = 0.498 \times 10^{-6}$$

$$\frac{1}{5} L_4 = 0.155$$

$$1000\, R_n = 500.$$

The network is shown in Fig. 8.

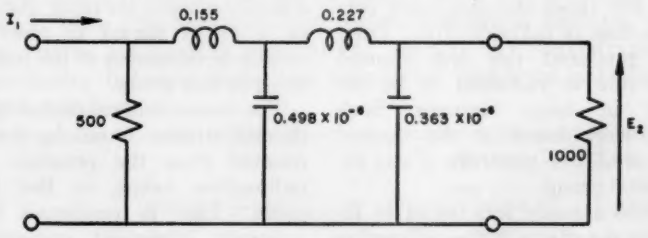


FIG. 8. Network obtained in Example 3.1 (values in ohms, henrys, and farads).

*(To be continued)*

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**Radioactivity Studies on Animals.**—Experiments conducted at the Hanford, Washington, plutonium plant over the past six years may provide valuable information to guide the use of radioactive isotopes in treating human ailments. General Electric scientists at Hanford have conducted a series of tests since 1950 to determine the maximum level of administration of the iodine isotope which would cause no damage to experimental animals. In the course of the experiment thyroid tumors developed in sheep which are considered to result from radioactive iodine consumed by the experimental animals.

In the experiments, researchers exposed some of the animals (sheep) to 300 to 500 times the maximum permissible dose of radioactivity. Their efforts produced the first thyroid tumors due to radiation to be described in large animals. Such tumors were found in the thyroid glands of all five members of one experimental group.

In 1950 a study was begun to investigate the effects of the radioactive iodine on animals when administered continuously in small quantities. The iodine isotope is widely used in clinical medicine. The sheep was chosen for the main study principally because it is the most important grazing animal in the neighborhood of the atomic project.

The experimental setup was so arranged that certain animals received the isotope only after reaching maturity. Most of the animals were offspring of ewes which were on the iodine feeding program during pregnancy. These offspring were exposed to the isotope before birth by means of the fetal circulation, then by way of the maternal milk during the nursing period and finally were fed the iodine in feed pellets after they were weaned at the age of four months. The exposure was carried out throughout the various periods in the animal's existence much as if the animal and its parent were grazing on land contaminated at a fairly constant level of radioactivity.

One sheep, sacrificed at the age of 56 months, had a large, malignant tumor. The other four animals, sacrificed at 68 months, contained many small benign tumors in their thyroid glands. Additional sheep treated in a similar manner are being maintained for a longer period to observe the further development of the lesions existing in this group.

The researchers concluded that the thyroid tumors found in the sheep resulted from the presence of the radioactive iodine in the thyroid gland. This is reinforced by the apparent infrequent occurrence of tumors of this type in sheep and absence of tumors in the thyroids of normal control animals.

Having concluded that radioactive iodine caused tumors, and cancer in one instance, the researchers took another look at the conditions of the experiment.

An important consideration in the development of tumors was prolonged and continuous exposure to the radiation. An additional significant factor was exposure of the experimental animals during early life.

# NOTE ON THE RELATIVISTIC HARMONIC OSCILLATOR

BY

LOUIS GOLD [1]

ABSTRACT

The relativistic harmonic oscillator is solved by an inverse fractional power series as has been done for the simple pendulum. The striking similarity of these two kinds of non-linear motion is thus demonstrated. The entire analysis is amply dealt with, without resort to elliptic integrals.

It was recently observed that the solution for the relativistic harmonic oscillator (r.h.o.) could be compared with that for the simple pendulum, particularly insofar as the periods of these two types of motion were concerned.[2] The period of the former was shown to be expressible in terms of elliptic integrals of both the first and second kind.

In this note a different manner of solving the r.h.o. is offered which has already been effectively utilized for the pendulum problem.[3] The need for elliptic integrals is avoided and, as will be demonstrated, the close similarity between these two non-linear problems is even more strikingly manifested.

Starting with the relativistic equation of motion for simple harmonic motion

$$\frac{d}{dt}(mv_x) = -kx, \qquad m = \frac{mo}{\sqrt{1 - v_x^2/c^2}} \tag{1}$$

and introducing the dimensionless parameter $\tau = \sqrt{\frac{k}{m_o}}\, t$, the problem is to solve

$$\left(\frac{d^2x}{d\tau^2}\right)^2 = x^2\left\{1 - K^2\left(\frac{dx}{d\tau}\right)^2\right\}^3, \qquad K = \frac{1}{c}\left(\frac{k}{m_o}\right)^{\frac{1}{2}} \tag{2}$$

subject to the boundary requirement that for $\tau = 0$, $x = a$ and $dx/d\tau = 0$.

Then, as in the pendulum problem,[3] a fractional power series of the form

$$\tau = f(s) = a_1 s^{\frac{1}{2}} + a_2 s^{\frac{3}{2}} + a_3 s^{\frac{5}{2}} + \cdots, \qquad s = a - x \tag{3}$$

[1] Massachusetts Institute of Technology, Cambridge, Mass.

[2] R. PENFIELD AND H. ZATZKIS, "The Relativistic Linear Harmonic Oscillator," JOUR. FRANKLIN INST., Vol. 262, p. 121 (1956).

[3] L. GOLD, "Solution for the Simple Pendulum Devoid of Elliptic Integrals," to be published.

is employed, whence the inverse differential equation for (2) becomes

$$[f''(s)]^2 = (a - s)^2\{[f'(s)]^2 - K^2\}^3. \tag{4}$$

The coefficients for (3) as derived in the Appendix are found to be

$$a_1 = \left(\frac{2}{a}\right)^{\frac{1}{2}}, \qquad a_2 = \frac{1}{3.2^3}\left(\frac{2}{a}\right)^{\frac{1}{2}}(1 + 3a^2K^2),$$
$$a_3 = \frac{3}{5}\cdot\frac{1}{2^7}\left(\frac{2}{a}\right)^{\frac{1}{2}}(1 + a^2K^2 + a^4K^4). \tag{5}$$

The solution for the r.h.o. may now be written as

$$\tau = \left(\frac{2}{a}\right)^{\frac{1}{2}}(a - x)^{\frac{1}{2}}\left\{1 + \frac{1}{3.2^3}\left(\frac{2}{a}\right)(1 + 3a^2K^2)(a - x) + \frac{3}{5.2^7}\left(\frac{2}{a}\right)^2(1 + 2a^2K^2 + 5a^4K^4)\cdot(a - x)^2 + \cdots\right\}. \tag{6}$$

Hence, since for the period $t = \frac{1}{4}T$, $T = 4\left(\frac{m_o}{k}\right)^{\frac{1}{2}}\tau$ and

$$T = 4\left(\frac{2}{a}\cdot\frac{m_o}{k}\right)^{\frac{1}{2}}(a - x)^{\frac{1}{2}}\left\{1 + \frac{1}{3.2^3}\left(\frac{2}{a}\right)(1 + 3a^2K^2)(a - x) + \frac{3}{5.2^7}\left(\frac{2}{a}\right)^2\cdot(1 + 2a^2K^2 + 5a^4K^4)(a - x)^2 + \cdots\right\}. \tag{7}$$

For the harmonic oscillator, the small amplitude limit $a \to 0$ has $T = 2\pi\left(\frac{m_o}{k}\right)^{\frac{1}{2}}$ and as

$$T_{a\to 0} = 4.2^{\frac{1}{2}}\left(\frac{m_o}{k}\right)^{\frac{1}{2}}s \tag{8}$$

then

$$s = \frac{\pi}{2}\cdot\frac{1}{2^{\frac{1}{2}}}. \tag{9}$$

This is precisely the finding for the simple pendulum[3] where

$$s = 1 + 2\sum_{n=1}^{\infty}\frac{2n - 1}{2n + 1}\cdot\frac{1}{2^{3n}}. \tag{10}$$

Indeed the form of (7) bears a striking resemblance to the general amplitude period for the pendulum.[3]

Reversion of (3) can be written down directly from the pendulum solution so that for

$$s = \tau(b_1\tau + b_2\tau^3 + b_3\tau^5 + \cdots) \tag{11}$$

one has the coefficients

$$b_1 = \frac{1}{a_1{}^2}, \qquad b_2 = -\frac{2a_2}{a_1{}^5}, \qquad b_3 = \frac{1}{a_1{}^8}(7a_2{}^2 - 2a_1a_3) \tag{12}$$

whereby the inverse appearance of (6) becomes

$$s = \frac{1}{2}a\tau\left\{\tau - \frac{1}{3.2^2}(1 + 3a^2K^2)\tau^3 + \frac{1}{3^2.5.2^4}(2 + 39a^2K^2 + 45a^4K^4)\tau^5 - \cdots\right\}. \tag{12}$$

Finally, the small amplitude limit gives the expected result for simple harmonic motion

$$x = a\left\{1 - \frac{1}{2!}\tau^2 + \frac{1}{4!}\tau^4 - \frac{1}{6!}\tau^6 + \cdots\right\} = a\cos\tau. \tag{13}$$

## APPENDIX

The utilization of the series solution $\tau = f(s)$ as given in (3) for the differential equation (4) requires the following quantities for the delineation of the $a_n$ coefficients:

$$f'(s) = \frac{1}{2}a_1s^{-1/2} + \frac{3}{2}a_2s^{1/2} + \frac{5}{2}a_3s^{3/2} + \frac{7}{2}a_4s^{5/2} + \frac{9}{2}a_5s^{7/2} + \cdots$$

$$[f'(s)]^2 = \frac{1}{4}a_1{}^2s^{-1} + \frac{3}{2}a_1a_2 + \left(\frac{9}{4}a_2{}^2 + \frac{5}{2}a_1a_3\right)s + \left(\frac{7}{2}a_1a_4 + \frac{15}{2}a_2a_3\right)s^2 + \cdots$$

$$|[f'(s)]^2 - K^2|^{\frac{1}{2}} = \frac{1}{64}a_1{}^6s^{-3} + \frac{3\lambda}{16}\cdot a_1{}^4s^{-2} + \left(\frac{3}{4}\lambda^2a_1{}^2 + \frac{3}{16}\cdot\frac{9}{4}a_1{}^4a_2{}^2 + \frac{3}{16}\cdot\frac{5}{2}a_1{}^6a_3\right)s^{-1} + \cdots$$

$$\lambda = \frac{3}{2}a_1a_2 - K^2$$

$$f''(s) = -\frac{1}{4}a_1s^{-3/2} + \frac{3}{4}a_2s^{-1/2} + \frac{15}{4}a_3s^{1/2} + \frac{35}{4}a_4s^{3/2} + \cdots$$

$$[f''(s)]^2 = \frac{1}{16}a_1{}^2s^{-3} - \frac{3}{8}a_1a_2s^{-2} + \left(\frac{9}{16}a_2{}^2 - \frac{15}{8}a_1a_3\right)s^{-1} + \cdots$$

The coefficients for the various powers of $s$ can now be written down

$$s^{-3}: \frac{1}{16}a_1{}^3 - \frac{1}{64}a_1{}^6a^2 = 0$$

$$s^{-2}: -\frac{3}{8}a_1a_2 - \frac{3}{16}\lambda a_1{}^4a^2 + \frac{2a}{64}a_1{}^6 = 0$$

$$s^{-1}: \left(\frac{9}{16}a_2{}^2 - \frac{15}{8}a_1a_3\right) - a^2\left(\frac{3}{4}\lambda^2a_1{}^2 + \frac{3}{16}\cdot\frac{9}{4}a_1{}^4a_2{}^2 + \frac{3}{16}\cdot\frac{5}{2}a_1{}^6a_3\right) + 2a\cdot\frac{3\lambda}{16}a_1{}^4 - \frac{1}{64}a_1{}^6 = 0.$$

From these equations the series coefficients stated in (5) are determinable.

**"Heat Barrier" Pushed Back in Field of Semiconductors.**—Experimental materials and techniques that promise to push back the "heat barrier" that now limits the performance of transistors and related electronic devices have been disclosed by the Radio Corporation of America.

Dr. Dietrich Jenny, research physicist at the David Sarnoff Research Center of RCA at Princeton, N. J., reported that experimental use of new compound semiconductors in place of germanium or silicon has led to laboratory types of transistors and diodes that can operate efficiently over a broad range of frequencies at temperatures as high as 850° F.—nearly 300° hotter than the maximum for present types.

The work, he said, is now leading into exploratory research that promises to result in further new compounds capable of operating efficiently at even higher temperatures—perhaps several hundred degrees higher.

Dr. Jenny emphasized that devices operating in the higher temperature range may have applications in electronic equipment for ultra-high-speed aircraft approaching the thermal barrier, and in further miniaturization of computers and other systems in which special cooling arrangements are now required. He called attention to the fact that germanium and silicon are the standard semiconductor materials used at the present time in transistors and diodes to perform a wide range of functions in electronic circuits. He explained that each of the two materials has its particular advantages—germanium being most useful for its high frequency properties, and silicon having the advantage of greater efficiency at higher temperatures.

Reporting on recent work at RCA Laboratories, Dr. Jenny said that efforts to combine in a single material the best features of both germanium and silicon have now resulted in new experimental devices employing various compound materials—notably gallium arsenide.

"We have every reason to believe that we can at least equal in a single material the high frequency properties of germanium and can work at much higher temperatures than the maximum for silicon," he said. "In the laboratory, using these materials, transistors have been made, and it is expected that future designs will live up to these theoretical expectations. Gallium arsenide microwave mixer and rectifier diodes of the point contact type have already yielded promising results compared with the equivalent germanium and silicon units."

He said that gallium arsenide microwave diodes capable of operation at radar frequencies in temperature environments well above the maximum for silicon units already have been made in substantial sample quantities in the laboratory. Furthermore, according to Dr. Jenny, gallium arsenide solar battery cells have shown promising performance in laboratory experiments. Silicon is used at present for solar batteries, which convert light radiation directly to electrical energy.

# ERRORS IN THE SOLUTION OF INTEGRAL EQUATIONS

BY

PARRY MOON [1] AND DOMINA EBERLE SPENCER [2]

SUMMARY

A novel method is developed for the solution of integral equations. An analytic solution is obtained by exponential approximation of the kernel; and to this solution is added the solution of an auxiliary integral equation. Since the auxiliary equation deals with magnitudes that are only a few per cent of the original quantities, a crude numerical solution of the auxiliary equation is satisfactory.

In fact, sufficient accuracy is obtained in many engineering applications by ignoring the auxiliary equation completely: all that is then needed is an indication of the maximum error caused by approximating the kernel. Simple equations are derived for the magnitude of this error. The results are applied to interflections in cylindrical blackbodies, and the errors are evaluated for previous solutions of the problem by Buckley and by Yamauti.

## 1. INTRODUCTION

For a number of engineering subjects, formulation in terms of integral equations is more direct and more easily visualized than the corresponding formulation in terms of differential equations. Unfortunately, however, methods of solving integral equations have lagged behind general theory. Mathematicians have been interested in existence proofs rather than in practical solutions, and textbooks on integral equations (1)[3] have mirrored this bias. In a few cases, the kernel of the integral equation is sufficiently simple so that analytic integration is feasible. But in most problems, simple analytic integration is impossible and analytic solutions cannot be obtained in closed form.

Faced by a problem of this nature, the investigator can always grind out numerical solutions (2); though such solutions have the great disadvantage that they possess no generality and must be worked out anew for each combination of parameters. Another approach is to employ a simple analytic approximation to the kernel. Kernel approximations have been used with considerable success, particularly for blackbody radiators (3,4) and for lighting design (5,6).

The solutions of integral equations with approximate kernels are obtained analytically in simple closed form; but these results are approximate because the true kernel has been replaced by an approximation. The question arises as to the magnitude of the error in the solution. This question does not seem to have been treated adequately.

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

It is true that Tricomi (7) developed an elegant expression for the error caused by approximating the kernel. But Tricomi's equation requires the evaluation of the reciprocal kernel; and a knowledge of the reciprocal kernel is essentially equivalent to a knowledge of the exact solution. Thus Tricomi's expression seems to have no practical value.

The purpose of this paper is to evaluate errors associated with approximations of the kernel. We first consider the general case of kernel approximation, then the exponential approximation, and finally a numerical example.

## 2. GENERAL CASE

This study deals with the Fredholm equation,

$$H(s) = H_o(s) + \rho \int_a^b K(s,t) \cdot H(t)\, dt, \tag{1}$$

where $\rho$, $H_o(s)$, and $K(s,t)$ are known and $H$ is to be determined. An approximate solution of Eq. 1 is obtained by replacing the true kernel $K(s,t)$ by an approximate kernel $K_a(s,t)$. Let

$$K(s,t) = K_a(s,t) + \Delta(s,t). \tag{2}$$

Also, let the true solution of Eq. 1 be

$$H(s) = H_a(s) + \delta(s), \tag{3}$$

where the approximate solution $H_a(s)$ satisfies the equation

$$H_a(s) = H_{oa}(s) + \rho \int_a^b K_a(s,t) \cdot H_a(t)\, dt. \tag{4}$$

In many engineering problems, $H_o = \text{const}$; in which case, no distinction need be drawn between $H_{oa}$ and $H_o$. But if $H_o$ is a function of $s$, one may find an analytic approximation advisable, with

$$H_o(s) = H_{oa}(s) + \zeta(s). \tag{5}$$

From Eq. 1,

$$H_a(s) + \delta(s) = H_{oa}(s) + \zeta(s) + \rho \left\{ \int_a^b K_a(s,t) \cdot H_a(t)\, dt + \int_a^b \Delta(s,t) \cdot H_a(t)\, dt + \int_a^b K(s,t) \cdot \delta(t)\, dt \right\}.$$

Subtraction of Eq. 4 gives the exact equation

$$\delta(s) = f(s) + \rho \int_a^b K(s,t) \cdot \delta(t)\, dt, \tag{6}$$

where

$$f(s) = \zeta(s) + \rho \int_a^b \Delta(s,t) \cdot H_a(t)\, dt. \tag{7}$$

Since $\zeta(s)$, $\Delta(s,t)$, and $H_a(t)$ are known, the function $f(s)$ is easily evaluated by numerical or graphical integration. The error $\delta(s)$ is then obtained by solution of Eq. 6. This is an integral equation of the same form as the original Eq. 1 and with the same kernel. The exact solution of Eq. 1 is thus the sum of the approximate solution $H_a(s)$ plus $\delta(s)$, in accordance with Eq. 3.

One may wonder if this method of solving an integral equation is not more trouble than the direct solution of Eq. 1 without approximation of the kernel. We believe, however, that the kernel-approximation method has real advantages. In the first place, $H_a(s)$ can usually be obtained within a few per cent of the true $H(s)$; and for many engineering applications, this is sufficient accuracy without considering $\delta(s)$. And if $\delta(s)$ is needed, it can be evaluated by very crude methods without appreciably affecting the accuracy of $H(s)$.

In some cases, it is sufficient to obtain a rough estimate of the error in $H_a(s)$ without solving Eq. 6. Evidently, several modifications of this procedure are possible.

*Approximation I.* Replace $f(s)$ by its maximum value $f_m$ in the interval $(a,b)$:

$$\delta(s) \leqq f_m + \rho \int_a^b K(s,t) \cdot \delta(t)\, dt. \tag{6a}$$

This equation is somewhat simpler than Eq. 6, so that an upper limit $\delta_m$ is easily obtained. The approximate kernel may be used instead of $K(s,t)$ if desired.

*Approximation II.* Replace $f(s)$ by $f_m$ and $\delta(t)$ by its maximum value $\delta_m$. Then

$$\delta_m \leqq f_m + \rho\delta_m \left[ \int_a^b K(s,t)\, dt \right]_{\max},$$

or

$$\delta_m \leqq \frac{f_m}{1 - \rho \left[ \int_a^b K(s,t)\, dt \right]_{\max}}. \tag{6b}$$

Equation 6*b* merely requires the evaluation of the area under the curve of $K(s,t)$ vs $t$. A few trials shows what value of $s$ will give a maximum area.

*Approximation III.* Replace $f(s)$ by $f_m$, $\delta(t)$ by $\delta_m$, and $K(s,t)$ by $K_m$. Then

$$\delta_m \leqq f_m + \rho\delta_m K_m \cdot (b - a)$$

or

$$\delta_m \leqq \frac{f_m}{1 - \rho K_m \cdot (b - a)}. \tag{6c}$$

In the important case, where $f_m \leqq \rho\Delta_m H_m(b - a)$ and $\zeta(s) = 0$, the ratio $(\delta_m/H_m)$ will not exceed the kernel discrepancy $(\Delta_m/K_m)$ if

$$2\rho(b - a)K_m \leqq 1. \tag{8}$$

This illustrates the effect of the parameter $\rho$ on the error. For small values of $\rho$, $H_a(s)$ is highly accurate. But as $\rho$ increases, the error caused by the approximate kernel becomes more pronounced.

### 3. EXPONENTIAL APPROXIMATIONS

Various analytic functions may be used in approximating the kernel, but the exponential function is simplest. If $K(s,t)$ and $H_a(s)$ are approximated, each by a series of exponentials, the approximate solution $H_a(s)$ may be expressed as a series of exponentials (3).

One common form of kernel is a function of $|s - t|$ and can be approximated by

$$K_a(s,t) = Ae^{-\alpha|s-t|}. \tag{9}$$

Then Eq. 4 becomes

$$H_a(s) = H_{oa}(s) + \rho A\left\{e^{-\alpha s}\int_a^s e^{\alpha t}H_a(t)\,dt + e^{\alpha s}\int_s^b e^{-\alpha t}H_a(t)\,dt\right\}. \tag{10}$$

If $H_{oa}(s) = H_o = \text{const}$, the solution is

$$H_a(s) = \frac{H_o}{1 - 2\rho A/\alpha}\left\{1 - \left(\frac{\alpha^2 - \beta^2}{\alpha}\right)\left[\frac{e^{-\beta s}}{M} + \frac{e^{\beta s}}{N}\right]\right., \tag{11}$$

where

$$M = (\alpha + \beta)e^{-\beta a} + (\alpha - \beta)e^{-\beta b},$$
$$N = (\alpha - \beta)e^{\beta a} + (\alpha + \beta)e^{\beta b},$$
$$\beta = \alpha(1 - 2\rho A/\alpha)^{\frac{1}{2}}.$$

For the special case of $a = 0$, $b \to \infty$, Eq. 11 reduces to

$$H_a(s) = \frac{H_o}{1 - 2\rho A/\alpha}\{1 - [1 - (1 - 2\rho A/\alpha)^{\frac{1}{2}}]e^{-\beta s}\}. \tag{11a}$$

For a cylinder of any finite length $l$, it is convenient to place the origin midway between the ends. Then $a = -l/2$, $b = +l/2$, and Eq. 11 becomes

$$H_a(s) = \frac{H_o}{1 - 2\rho A/\alpha}\left\{1 - \frac{2\rho A \cosh \beta s}{\alpha \cosh(\beta l/2) + \beta \sinh(\beta l/2)}\right\}. \quad (11b)$$

The foregoing equations give approximate solutions of the integral equation. The discrepancy $\delta(s)$ may be evaluated exactly by use of Eq. 6, where an iteration process is employed with graphical integration. Or a close approximation may be obtained by replacing $K(s,t)$ by $K_a(s,t)$. Then Eq. 6 becomes

$$\delta(s) \cong f(s) + \rho A\left\{e^{-\alpha s}\int_a^s e^{\alpha t}\delta(t)\,dt + e^{\alpha s}\int_a^b e^{-\alpha t}\delta(t)\,dt\right\}. \quad (12)$$

Usually even cruder approximations are permissible:

*Approximation I.* Replace $f(s)$ by $f_m$. Then Eq. 12 is of the same form as Eq. 10 and its solution is

$$\delta(s) \leqq \frac{f_m}{1 - 2\rho A/\alpha}\left\{1 - \left(\frac{\alpha^2 - \beta^2}{\alpha}\right)\left[\frac{e^{-\beta s}}{M} + \frac{e^{\beta s}}{N}\right]\right\}. \quad (13)$$

For limits $a = 0$, $b \to \infty$,

$$\delta(s) \leqq \frac{f_m}{1 - 2\rho A/\alpha}\{1 - [1 - (1 - 2\rho A/\alpha)^{\frac{1}{2}}]e^{-\beta s}\}, \quad (13a)$$

and the maximum discrepancy is

$$\delta_m \leqq \frac{f_m}{1 - 2\rho A/\alpha}. \quad (14)$$

For limits $a = -l/2$, $b = +l/2$,

$$\delta(s) \leqq \frac{f_m}{1 - 2\rho A/\alpha}\left\{1 - \frac{2\rho A \cosh \beta s}{\alpha \cosh(\beta l/2) + \beta \sinh(\beta l/2)}\right\} \quad (13b)$$

and the maximum discrepancy is

$$\delta_m \leqq \frac{f_m}{1 - 2\rho A/\alpha}\left\{1 - \frac{2\rho A/\alpha}{\cosh(\beta l/2) + (\beta/\alpha)\sinh(\beta l/2)}\right\}. \quad (15)$$

*Approximation II.* Replace $f(s)$ by $f_m$ and $\delta(t)$ by $\delta_m$:

$$\delta(s) \leqq f_m + \rho A \delta_m\left\{e^{-\alpha s}\int_a^s e^{\alpha t}\,dt + e^{\alpha s}\int_s^b e^{-\alpha t}\,dt\right\}$$

$$\leqq f_m + \frac{2\rho A \delta_m}{\alpha} \{1 - \tfrac{1}{2}[e^{-\alpha(s-a)} + e^{-\alpha(b-s)}]\}. \tag{16}$$

For $a = 0$, $b \to \infty$,

$$\delta(s) \leqq f_m + \frac{2\rho A \delta_m}{\alpha} (1 - e^{-\alpha s}/2) \tag{16a}$$

and $\delta_m$ is expressed by Eq. 14. For $a = -l/2$, $b = +l/2$,

$$\delta(s) \leqq f_m + \frac{2\rho A \delta_m}{\alpha} [1 - e^{-\alpha l/2} \cosh \alpha s] \tag{16b}$$

and $\delta_m$ is again given by Eq. 14.

We have derived a number of equations for error, to meet the wide variety of conditions that are encountered in the engineering application of integral equations. The crudest criterion for $\delta_m$ is given by Approximation III, Eq. 6*c*. Obviously, this criterion is worthless if $(b - a)$ is large. If Eq. 6*c* is inapplicable, try Eq. 14 or 15. For progressively greater refinement, we have Eqs. 13, 12, and 6. The quantity $f_m$ may also be approximated. If $\Delta(s,t)$ is replaced by $\Delta_m$, Eq. 7 becomes

$$f_m \leqq \zeta_m + \rho \Delta_m \int_a^b H_a(t)\, dt. \tag{7a}$$

And if $\Delta(s,t)$ is replaced by $\Delta_m$ and $H_a(t)$ is replaced by $H_m$,

$$f_m \leqq \zeta_m + \rho \Delta_m H_m (b - a). \tag{7b}$$

#### 4. INTERFLECTIONS IN CYLINDERS

The problem of multiple reflections of radiation is one of the engineering applications of Eq. 1. The walls of the enclosure are perfectly diffusing, and the geometry is such that $H$ is expressible as a function of the single variable $s$. In this application, $H(s)$ refers to either radiometric or photometric quantities (**6**). The initial helios, without interflections, is denoted by $H_o(s)$ and the helios including all interflections is $H(s)$. The reflectance of the surface is $\rho$.

For a circular cylinder, $s$ and $t$ are distances along the axis, normalized in terms of the radius, and the true kernel is

$$K(s,t) = \frac{1}{2}\left[1 - \frac{|s - t|[(s - t)^2 + 6]}{[(s - t)^2 + 4]^{\frac{3}{2}}}\right]. \tag{17}$$

For a cylinder of rectangular cross section, the kernel is expressed in a somewhat more complicated form, Eq. 6 of a previous paper (**8**). In

either case, the approximate kernel is given by Eq. 9 of the present paper; and

$$\alpha = \frac{pw}{2S'} \qquad A = \alpha/2,$$

where

$p$ = perimeter of the cross section,
$w$ = width of rectangular cross section (or radius of circle),
$S$ = cross-sectional area.

Thus for a circular cylinder with $H_o$ = const, the integral equation becomes

$$H_a(s) = H_o + \rho \left\{ e^{-s} \int_a^s e^t H_a(t)\, dt + e^s \int_s^b e^{-t} H_a(t)\, dt \right\}. \qquad (18)$$

For limits $a = 0$, $b \to \infty$, the solution is

$$H_a(s) = \frac{H_o}{1-\rho} \left[ 1 - (1 - \sqrt{1-\rho}) e^{-\sqrt{1-\rho}\, s} \right]. \qquad (19)$$

For $a = -l/2$, $b = +l/2$,

$$H_a(s) = \frac{H_o}{1-\rho} \left[ 1 - \frac{\rho \cosh \beta s}{\cosh(\beta l/2) + \sqrt{1-\rho} \sinh(\beta l/2)} \right] \qquad (20)$$

where $\beta = (1-\rho)^{\frac{1}{2}}$. Similar expressions for the rectangular case are given by Eqs. 11*a* and 11*b*. Errors caused by kernel approximation are evaluated by the formulas of Section 3.

### 5. A NUMERICAL EXAMPLE

As an example, consider interflections in a circular cylinder extending from $s = 0$ to $s \to \infty$. This problem was solved by Buckley (3) and by Yamauti (4), using one-, two-, and three-term exponential approximations to the kernel. Yamauti apparently believed that his three-term approximation gave the true result, for he used this solution to determine the lengths of cylindrical radiators to give 0.1 per cent departure from true blackbody radiation. But there is no justification for such an assumption of zero error. Thus it is interesting to apply our equations to this problem, particularly since it is not without practical significance.

The true kernel is given by Eq. 17, while the single-term exponential approximation is expressed by Eq. 9 with $A = \frac{1}{2}$, $\alpha = 1$. The difference between the two is plotted in Fig. 1 and the values are listed in Table I.

The approximate solution $H_a(s)$ is easily evaluated from Eq. 19 for any $H_o$ and any reflectance $\rho$. Obviously, the discrepancy between the true solution and the approximate solution will be largest for high reflectance. We have chosen $\rho = 0.90$, the highest value (and there-

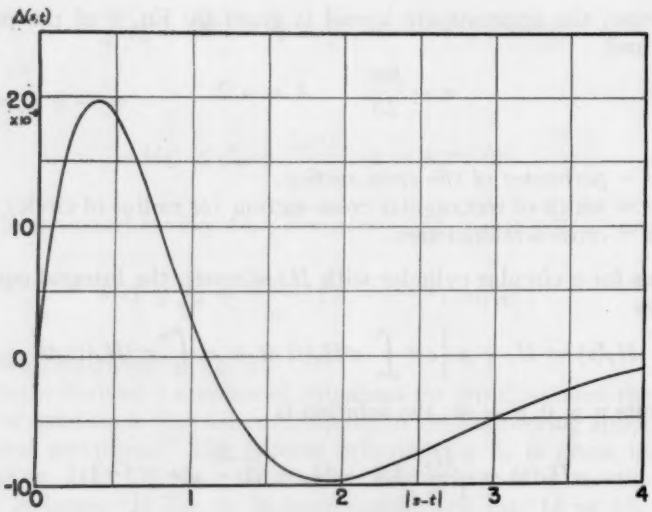


FIG. 1. Kernel discrepancy, $\Delta(s,t) = K(s,t) - K_a(s,t)$ for circular cylinder.

TABLE I.—*True and Approximate Kernels for Circular Cylinders.*

| $\lvert s-t\rvert$ | $K(s,t)$ | $K_a(s,t)$ | $\Delta(s,t)$ |
|---|---|---|---|
| 0.0 | $^{0}50000$ | $^{0}50000$ | 0 |
| 0.5 | 32167 | 30326 | $+18.41\times10^{-3}$ |
| 1.0 | 18695 | 18394 | +3.01 |
| 1.5 | $^{0}10400$ | $^{0}11156$ | −7.56 |
| 2.0 | $^{1}58058$ | $^{1}67668$ | −9.61 |
| 2.5 | $^{1}33384$ | 41043 | −7.66 |
| 3.0 | 19972 | 24894 | −4.92 |
| 3.5 | $^{1}12449$ | $^{1}15099$ | −2.65 |
| 4.0 | $^{2}80655$ | $^{2}91578$ | −1.09 |
| 4.5 | 54113 | 55545 | −0.143 |
| 5.0 | $^{2}37442$ | $^{2}33690$ | +0.375 |
| 5.5 | 26641 | 20434 | 0.621 |
| 6.0 | $^{2}19416$ | $^{2}12394$ | 0.702 |
| 7.0 | $^{2}10960$ | $^{3}45594$ | 0.640 |
| 8.0 | $^{3}66190$ | $^{3}16773$ | 0.494 |
| 9.0 | 42192 | $^{4}61705$ | 0.360 |
| 10.0 | 28107 | 22700 | 0.258 |
| 20.0 | $^{4}18411$ | $^{5}10306$ | 0.018 |
| ∞ | 0 | 0 | 0 |

NOTE—The superscripts refer to number of zeros after decimal point. For instance, $^{2}80655 = 0.0080655$.

fore the most troublesome case) that ordinarily obtains in engineering practice. Without loss of generality, we also choose $H_o = 0.10$. For any other value of $H_o$, linearity insures that the values of $H(s)$ will be proportional to $H_o$ and the per cent errors will be unaltered.

TABLE II.—*$f(s)$ for Semi-Infinite Circular Cylinder with $\rho = 0.90$ and $H_o = 0.10$.*

| $s$ | $f(s)$ |
|---|---|
| 0 | $(-2.14 \pm 0.01) \times 10^{-3}$ |
| 0.5 | +0.55 |
| 1.0 | 3.33 |
| 1.5 | 4.05 |
| 2.0 | 3.48 |
| 2.5 | 2.44 |
| 3.0 | 1.52 |
| 3.5 | +0.69 |
| 4.0 | −0.061 |
| 4.6 | −0.657 |
| ∞ | 0.000 |

The next step is to evaluate $f(s)$ from Eq. 7. The integration was effected by plotting the products $\Delta \cdot H_o$ and measuring the area with a planimeter. The results (Table II) are believed to be correct to $\pm 10^{-5}$, which is 0.001 per cent of the maximum value of $H(s)$. A curve of $f(s)$ is shown in Fig. 2.

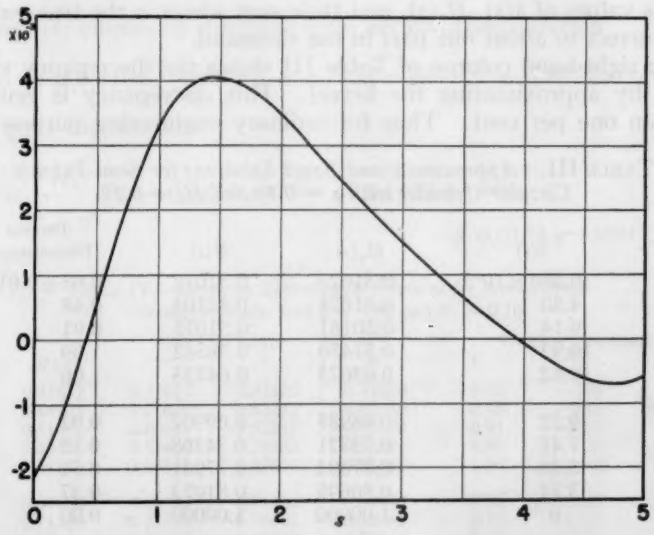


FIG. 2. $f(s)$ for semi-infinite circular cylinder with $\rho = 0.90$ and $H_o = 0.10$.

Values of $f(s)$ were then substituted into Eq. 6, and this integral equation was solved by iteration. The resulting $\delta(s)$ is plotted in Fig. 3, the values being correct to within approximately $\pm 10^{-4}$. Table

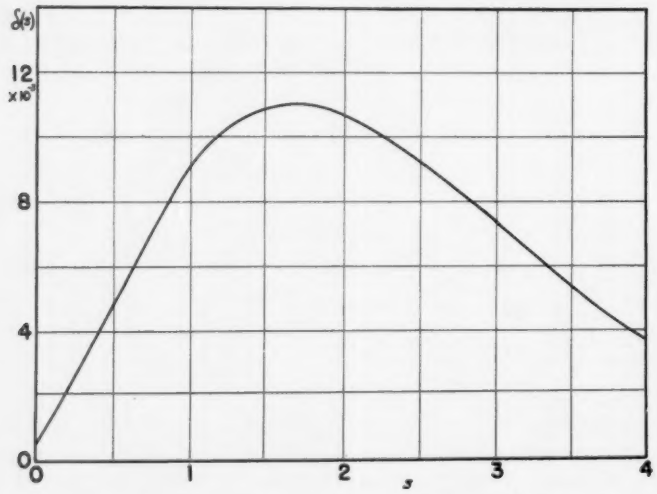


FIG. 3. Discrepancy between exact and approximate solutions of the integral equation. Semi-infinite circular cylinder with $\rho = 0.90$ and $H_o = 0.10$.

III lists values of $\delta(s)$, $H_a(s)$, and their sum which is the true value of $H(s)$, correct to about one part in ten thousand.

The right-hand column of Table III shows the discrepancy that is caused by approximating the kernel. This discrepancy is generally less than one per cent. Thus for ordinary engineering purposes, *the*

TABLE III.—*Approximate and Exact Solutions for Semi-Infinite Circular Cylinder with $\rho = 0.90$ and $H_o = 0.10$.*

| $s$ | $\delta(s)$ | $H_a(s)$ | $H(s)$ | Percent Discrepancy* |
|---|---|---|---|---|
| 0.0 | $0.486 \times 10^{-3}$ | 0.31623 | 0.31672 | $0.05 \pm 0.01\%$ |
| 0.5 | 4.80 | 0.41624 | 0.42104 | 0.48 |
| 1.0 | 9.14 | 0.50161 | 0.51075 | 0.91 |
| 1.5 | 10.92 | 0.57450 | 0.58542 | 1.09 |
| 2.0 | 10.62 | 0.63673 | 0.64735 | 1.06 |
| 2.5 | 9.22 | 0.68985 | 0.69907 | 0.92 |
| 3.0 | 7.47 | 0.73521 | 0.74268 | 0.75 |
| 3.5 | 5.38 | 0.77393 | 0.77931 | 0.54 |
| 4.0 | 3.74 | 0.80699 | 0.81073 | 0.37 |
| $\infty$ | 0 | 1.00000 | 1.00000 | 0.00 |

* Difference between true solution $H(s)$ and approximate solution $H_a(s)$, expressed in per cent of maximum value of $H(s)$.

*analytic solution*, Eq. 19, *obtained with the approximate kernel is amply precise.* This is particularly true with the usual values of $\rho < 0.90$, which will reduce $\delta(s)$ even further.

Equation 14 provides a simple way of obtaining an upper limit to the discrepancy $\delta(s)$ without solving the auxiliary integral equation. According to Fig. 2, $f_m \cong 40 \times 10^{-3}$, so Eq. 14 gives

$$\delta_m \leqq \frac{f_m}{1-\rho} = 10 f_m = 40 \times 10^{-3}.$$

This says that the discrepancy caused by approximating the kernel never exceeds 4 per cent of the maximum value of $H(s)$. Table III shows that the actual discrepancy is one per cent. Like most of these approximate expressions, the criterion of Eq. 14 is a coarse one, though it may be useful if one does not care to investigate the problem more carefully.

Buckley (**3**) assumed that a one-term approximation is not sufficiently accurate and used

$$K_a(s,t) = \tfrac{1}{2}[1.21\, e^{-1.14|s-t|} - 0.21\, e^{-3.40|s-t|}],$$

which gives the solution

$$H_a(s) = \frac{H_o}{1-\rho}[1 - 0.6895\, e^{-0.3518s} + 0.0057\, e^{-3.484s}]. \qquad (21)$$

Yamauti (**4**) employed the three-term kernel approximation

$$K_a(s,t) = \tfrac{1}{2}[0.1104\, e^{-0.5363|s-t|} + 1.7123\, e^{-1.495|s-t|} - 0.8227\, e^{-2.343|s-t|}]$$

which gave the solution

$$H_a(s) = \frac{H_o}{1-\rho}[1 - 0.5825\, e^{-0.2775s} - 0.1167\, e^{-0.8327s} + 0.0154\, e^{-2.5705s}]. \qquad (22)$$

TABLE IV.—*Errors in Approximate Solutions for Semi-Infinite Circular Cylinder with* $\rho = 0.90$ *and* $H_o = 0.10$.

| | | $H_a(s)$ | | | Per Cent Discrepancy | | |
|---|---|---|---|---|---|---|---|
| $s$ | $H(s)$ | 1. | 2. | 3. | 1. | 2. | 3. |
| 0.0 | 0.31672 | 0.31623 | 0.31620 | 0.31620 | +0.05 | +0.05 | +0.05 |
| 0.5 | 0.42104 | 0.41624 | 0.42272 | 0.42030 | 0.48 | −0.17 | 0.07 |
| 1.0 | 0.51075 | 0.50161 | 0.51517 | 0.50908 | 0.91 | −0.44 | 0.17 |
| 1.5 | 0.58542 | 0.57450 | 0.59325 | 0.58271 | 1.09 | −0.78 | 0.27 |
| 2.0 | 0.64735 | 0.63673 | 0.65884 | 0.64362 | 1.06 | −1.15 | 0.37 |
| 2.5 | 0.69907 | 0.68985 | 0.71386 | 0.69442 | 0.92 | −1.48 | 0.47 |
| 3.0 | 0.74268 | 0.73521 | 0.76002 | 0.73705 | 0.75 | −1.73 | 0.56 |
| 3.5 | 0.77931 | 0.77393 | 0.79873 | 0.77314 | 0.54 | −1.94 | 0.62 |
| 4.0 | 0.81073 | 0.80699 | 0.83119 | 0.80385 | 0.37 | −2.05 | 0.69 |

Calculated values are listed in Table IV, where the column headings 1, 2, 3 refer to one-term, two-term, and three-term kernel approximations. These values are compared with the true $H(s)$ of Table III and the errors are calculated. Figure 4 shows that the one-term

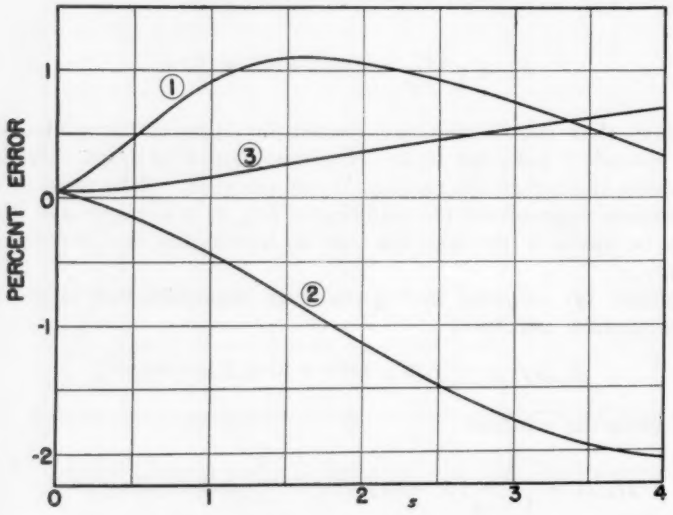


FIG. 4. Errors in approximate solutions for the cylindrical blackbody radiator.

1, one-term approximation to the kernel,
2, two-term approximation,
3, three-term approximation.

approximation, advocated in this paper, compares very favorably with the more complex expressions employed by Buckley and Yamauti.

## CONCLUSIONS

1. A promising method of obtaining analytic solutions of integral equations is by one-term exponential approximations of the kernels.

2. The accuracy of the approximate solution may be estimated by use of Eq. 14 or similar equations given in the paper.

3. If higher accuracy is required, the auxiliary Eq. 6 may be solved to obtain $\delta(s)$, which is added to the approximate solution $H_a(s)$ to obtain the true solution $H(s)$.

4. For the interflection problem in cylinders, there seems to be no advantage in using more than one exponential in approximating the kernel.

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**Titanium Furnace Controlled with Atomic Radiations.**—Atomic radiations, in the form of highly energetic gamma rays from radioactive cobalt, have been put to a new and important use by scientists of the Westinghouse Research Laboratories. Dr. Daniel Alpert, associate director of the Laboratories, stated that the gamma rays are being employed to detect and control the level of molten titanium in a new "cold hearth" arc furnace designed to prepare the purified titanium metal. The titanium furnace on which the control has been placed is a major installation at the Westinghouse metals development plant at Blairsville, Pa. To carry out their new assignment, the penetrating gamma rays pass through as much as 15 inches of solid metal.

"The successful operation of this new titanium arc furnace demands exact control of the molten surface of the titanium ingot inside it," Dr. Alpert said. "Such control must be foolproof, precise and fast-working—yet must be accomplished under extremely difficult conditions. For example, the temperature of the white-hot liquid titanium is more than 3000° F.—made so by an electric arc which consumes as much current as the combined capacity of 150 modern homes.

"Gamma rays are penetrating, high-energy radiations which come from the spontaneous disintegration of certain radioactive atoms," Dr. Alpert continued. "By beaming these rays through the walls of the furnace and the 12-in. titanium ingot inside it, we are able to 'see' the actual level of the titanium. We use this information to automatically raise or lower the ingot to its correct operating level, thereby insuring proper operation of the furnace.

"This control by gamma rays, we believe, brings a new degree of reliability and safety to titanium arc furnace operation. It is a critical job of measurement and control which would be very difficult to handle by any other means."

The gamma rays passing through the furnace are detected by means of two "scintillation counters." Such a counter contains a crystal which changes the gamma rays into flashes of light. The counter then converts the light flashes into electrical pulses and amplifies them. These amplified pulses are fed to electronic circuits which are designed to drive a hydraulic system that raises or lowers the titanium ingot to the exact position required for proper operation of the arc furnace.

The whole system is so precise that it can detect and maintain the level of the titanium ingot to within one-hundredth of an inch of its ideal operating position. If, for any reason, the ingot moves beyond its prescribed limits, the gamma ray control causes immediate shutdown of the furnace.

Although designed expressly for the Westinghouse titanium furnace installed at Blairsville, the gamma ray control could be used on other types of furnaces and in other position-measuring and leveling applications.

# SHEAR BUCKLING OF SIMPLY SUPPORTED PLATES TAPERED IN PLANFORM

BY

BERTRAM KLEIN [1]

SUMMARY

A simple, approximating set of equations is derived for calculating the shear buckling loads of isosceles trapezoidal plates. A numerical example is worked for a typical plate. The result appears to be good.

## LIST OF SYMBOLS

$a$ = plate length
$b$ = plate width (variable)
$b_1$ = minimum plate width
$c_n$ = deflected shape coefficient
$D$ = plate bending rigidity
$f$ = function
$K$ = constant
$n$ = 1,2,3
$q$ = shear flow (variable)
$q_1$ = maximum shear flow
$w$ = plate normal deflection
$x,y$ = rectangular coordinates
$x_1$ = coordinate measured from edge of plate
$\alpha = \frac{x_1}{a}$
$\beta$ = deflected shape parameter
$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$
$\lambda$ = eigenvalue
$\phi$ = angle
$\psi$ = angle
$\theta$ = semi-angle of plate taper

## INTRODUCTION

Little attention in the literature has been given the problem of the buckling of tapered plates. Previously the author has considered the compressive buckling of plates tapered in planform.[2] The purpose of the present endeavor is to solve the problem of the shear buckling of such plates. The plate and its loading are depicted in Fig. 1. As seen in this figure, the plate is assumed to have an axis of symmetry, that is, the $x$ axis. In order to maintain equilibrium, the applied shear flow varies quadratically in the $x$ direction such that

$$qb^2 = q_1 b_1^2 = \text{const.}, \tag{1}$$

it being assumed to be constant in the $y$ direction. The edges of the plate are assumed to be simply supported, and the material is assumed to behave elastically under load.

The problem is solved by the approximate method of collocation.[3] Use of this method can be shown to yield accurate solutions to com-

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[2] B. KLEIN, "Buckling of Simply Supported Plates Tapered in Planform," *J. Appl. Mech.*, Vol. 23, pp. 207–213 (1956).

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plicated physical problems with relatively little laborious calculations provided

1. the physical behavior of the system is known approximately, and
2. the choice of required functions and regions of collocation is good.

If the above knowledge is not at hand, accurate results by using the method still are possible, but the amount of labor becomes too excessive to make the method practical.

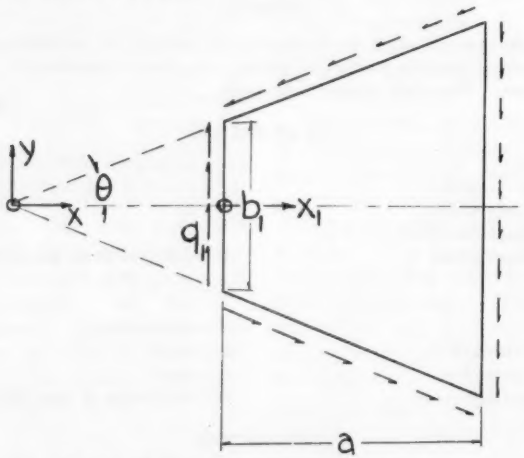


FIG. 1. Tapered plate geometry and loading.

### METHOD OF SOLUTION

The fundamental differential equation in rectangular coordinates defining the shear buckling of constant thickness thin plates is

$$D\Delta\Delta w - 2q\frac{\partial^2 w}{\partial x \partial y} = 0. \tag{2}$$

In the present instance the deflected shape of the plate is taken in the form

$$w = \sum_{n=1} \sum_{\beta} c_{n\beta}\left[\sin\left(\frac{n\pi x_1}{a}\right) - n\beta \sin\left(\frac{\pi y \cot\theta}{2x}\right)\sin\frac{\pi x_1}{a}\right] \times \cos\left(\frac{\pi y \cot\theta}{2x}\right) \tag{3}$$

$$= \sum \sum c_{n\beta} \sin\phi \cos\psi$$

where

$$\frac{\cot\theta}{2x} = \frac{1}{b}. \tag{3a}$$

$\beta$ is a parameter related to the orientation of twist of the plate deflected surface. The expression given in Eq. 3 satisfies deflection boundary conditions exactly but moment boundary conditions only at certain points. However since this deflected shape can be made to represent the general physical behavior of the plate, and points chosen for collocation are to be in appropriate regions, accurate solutions are possible in spite of some relaxation of the moment boundary conditions.

Points chosen for collocation are along the $x$ axis, that is, $y = 0$. This choice results in considerable simplification besides being a natural one. The necessary partial derivatives of $w$ are calculated from formulae derived in the appendix and are at $y = 0$:

$$\frac{\partial^4 w}{\partial x^4} = \left(\frac{n\pi}{a}\right)^4 \sin n\alpha, \qquad \alpha = \frac{\pi x_1}{a} \tag{4a}$$

$$\begin{aligned}\frac{\partial^4 w}{\partial x^2 \partial y^2} = {} & 4\,\frac{n\pi}{a}\left(\frac{\pi}{b}\right)^2 (n\beta)^2 \left(\frac{1}{x}\sin\alpha - \frac{\pi}{a}\cos\alpha\right)\sin\alpha\cos n\alpha \\ & + \left(\frac{\pi}{b}\right)^2 (n\beta)^2 \left\{\left[2\left(\frac{\pi^2}{a^2} - \frac{2}{x^2}\right) + n^2\frac{\pi^2}{a^2}\right]\sin^2\alpha \right. \\ & \left. + 2\,\frac{\pi}{ax}\sin 2\alpha - 2\left(\frac{1}{x}\sin - \frac{\pi}{a}\cos\alpha\right)^2\right\}\sin n\alpha \\ & + \left(\frac{\pi}{b}\right)^2\left(\frac{n\pi}{a}\right)^2 \sin n\alpha + 4\left(\frac{\pi}{b}\right)^2 \frac{1}{x}\,\frac{n\pi}{a}\cos n\alpha \\ & - 6\left(\frac{\pi}{b}\right)^2 \frac{1}{x^2}\sin n\alpha \end{aligned} \tag{4b}$$

$$\frac{\partial^4 w}{\partial y^4} = \left[\left(\frac{\pi}{b}n\beta\sin\alpha\right)^4 + 10\left(\frac{\pi}{b}n\beta\sin\alpha\right)^2 + \left(\frac{\pi}{b}\right)^4\right]\sin n\alpha \tag{4c}$$

$$\frac{\partial^2 w}{\partial x \partial y} = n\beta\left(\frac{\pi}{b}\right)\left[\left(\frac{1}{x}\sin\alpha - \frac{\pi}{a}\cos\alpha\right)\cos n\alpha + \frac{n\pi}{a}\sin\alpha\sin n\alpha\right]. \tag{4d}$$

Substitution of $\frac{1}{x} = \frac{2\tan\theta}{b}$ into Eqs. 4 and the results into Eq. 2, multiplying by $\frac{1}{D}\left(\frac{b}{\pi}\right)^4$, and collecting terms yields for each $c_{n\beta}$,

$$\begin{aligned}\left\{\left[\left(\frac{nb}{a}\right)^2 + 1\right]^2 - K_1 + (n\beta)^2\left[\left(-K_1 + 2\left(\frac{nb}{a}\right)^2 + 10\right)\sin^2\alpha \right.\right. \\ \left.\left. + \left(\frac{b}{a}\right)K_2\sin 2\alpha - \left(\frac{2b}{a}\right)^2\cos 2\alpha\right] + (n\beta)^4\sin^4\alpha\right\}\sin n\alpha +\end{aligned}$$

*Continued on next page*

$$+\left[n\left(\frac{b}{a}\right)^2(n\beta)^2\left(\frac{a}{b}K_2\sin\alpha-8\cos\alpha\right)\sin\alpha+n\left(\frac{b}{a}\right)K_2\right]\cos n\alpha$$

$$-2\lambda\left(\frac{b}{a}\right)(n\beta)\left[\frac{1}{8}\left(\frac{a}{b}K_2\sin\alpha-8\cos\alpha\right)\cos n\alpha+n\sin\alpha\sin n\alpha\right]=0 \quad (5)$$

where

$$K_1=\frac{48}{\pi^2}\tan^2\theta, \qquad K_2=\frac{16}{\pi}\tan\theta \quad (5a)$$

and

$$\lambda=\frac{qb^2}{D\pi^2}. \quad (5b)$$

Equations 5 define the problem. The solution for $\lambda$ is one of determining the lowest eigenvalue of a matric equation. The size of the matrix is equal to the number of points chosen for collocation which equals the number of undetermined coefficients, $c_{n\beta}$. The two basic parameters defining a particular plate are $\theta$, and $b_1/a$.

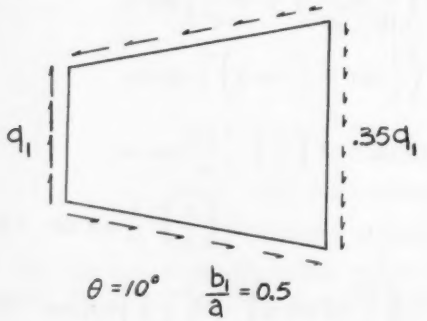


FIG. 2. Typical tapered plate used in numerical example.

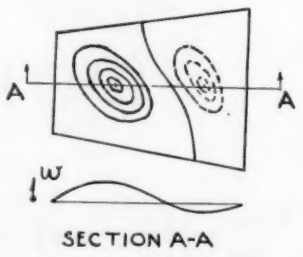


FIG. 3. Approximate deflected shape of plate in numerical example.

## NUMERICAL EXAMPLE

A typical plate tapered in planform is shown in Fig. 2. Three points along the $x$ axis are chosen for collocation as

$$\frac{x_1}{a}=\tfrac{1}{3},\ \tfrac{1}{2}\text{ and }\tfrac{2}{3}. \quad (6)$$

Values of $\beta$ are taken as follows

| $n$ | $\beta$ |
|---|---|
| 1 | 0.75 |
| 2 | 0.5 |
| 3 | 0.4 |

(7)

The choice of these values of $\beta$ is based upon calculations carried out for the shear buckling of non-tapered plates buckling into one, two and three waves, respectively. Evaluation of Eq. 5 for the chosen data yields the matric equation:

$$\begin{array}{c|ccc} \frac{x_1}{a} \backslash n & 1 & 2 & 3 \\ \hline \frac{1}{3} & (6.18-0.535\lambda) & (15.74-2.070\lambda) & (2.27-0.512\lambda) \\ \frac{1}{2} & (9.37-1.12\lambda) & (-2.42+0.222\lambda) & (-58.86+4.86\lambda) \\ \frac{2}{3} & (5.61-0.624\lambda) & (-20.69+2.475\lambda) & (7.945-0.650\lambda) \end{array} = 0. \quad (8)$$

Solution yields

$$\lambda = 8.20. \quad (8a)$$

It is further found that

$$c_2 = 1.33c_1 \quad \text{and} \quad c_3 = 0.013c_1. \quad (8b)$$

The major deflection occurs in the region of higher shear stress which is to be expected. A sketch of the approximate deflected shape is shown in Fig. 3. This shape appears to be the correct one for the structure and loading considered.

In a manner similar to the procedure carried out in the above numerical example, the shear buckling loads of other tapered plates may be calculated. However, it should be pointed out that for plates with relatively large values of $\theta$ a nodal line may occur approximately in the direction of the $x$ axis. In such cases the expression for $w$ given in Eq. 3 is not valid.

## APPENDIX

Basic partial derivatives needed in deriving Eqs. 4 are listed here.

Consider

$$\sin f(x, y) \equiv \sin \phi$$

$$\frac{\partial \sin\phi}{\partial x} = f_x \cos\phi; \qquad \frac{\partial \sin\phi}{\partial x \partial y} = f_{xy}\cos\phi - f_x f_y \sin\phi; \qquad \frac{\partial^2 \sin\phi}{\partial x^2} = -f_x^2 \sin\phi + f_{xx}\cos\phi$$

$$\frac{\partial^3 \sin\phi}{\partial x^2 \partial y} = (f_{xxy} - f_x^2 f_y)\cos\phi + (-f_{xx}f_y - 2f_x f_{xy})\sin\phi$$

$$\frac{\partial^3 \sin\phi}{\partial x^3} = (-f_x^3 + f_{xxx})\cos\phi + (-3f_x f_{xx})\sin\phi$$

$$\frac{\partial^4 \sin\phi}{\partial x^2 \partial y^2} = -(f_x^2 f_{yy} - f_{xxyy} + 4f_x f_y f_{xy} + f_y^2 f_{xx})\cos\phi + (f_x^2 f_y^2 - 2f_{xy}^2 - f_{xx}f_{yy} - 2f_{xxy}f_y - 2f_{yyx}f_x)\sin\phi$$

$$\frac{\partial^4 \sin\phi}{\partial x^4} = (f_{xxxx} - 6f_x^2 f_{xx})\cos\phi + (f_x^4 - 4f_x f_{xxx} - 3f_{xx}^2)\sin\phi.$$

Derivatives with respect to $y$ are obtained by replacing $x$ by $y$ in the above equations. Derivatives of $\cos f(x,y)$ are obtained by replacing $\cos\phi$ by $-\sin\phi$ and $\sin\phi$ by $\cos\phi$ in the above equations.

**High-Strength Staple Fiber.**—Avisco XL, a new, high-strength staple fiber, has been announced by American Viscose Corporation. Characterized as the greatest improvement in rayon since its invention, yarns in the new staple are 70 per cent stronger when wet and 40 per cent stronger in the dry-conditioned state then conventional rayons.

Dr. Herschel H. Cudd, Vice President in charge of Research and Development, indicated that the new fiber has broad market applications in industrial uses as well as in apparel and home-furnishings. "In comparable construction and weave," he said, "Avisco XL fabrics have 25 per cent more breaking strength and 33 per cent more tear strength than cotton." This predicts greater serviceability in industrial ducks, tarps, and similar fabrics.

Avisco XL has been spun at American Viscose's Textile Research Department into yarns as fine as 60/1 with 1.5 denier, $1\frac{9}{16}$" staple. Because of increased extensility, the new fiber is compatible with the acrylics and polyesters (the non-cellulosic fibers) for blends. This indicates new, lighter-weight, minimum care fabrics with the draping characteristics of regular rayon.

The product has been in successful pilot plant production for over six months and now limited commercial quantities are available for selected end uses.

**Fort Belvoir Labs Grow 32-Pound Crystal.**—Chemists at the Army Engineers' Research and Development Laboratories, Fort Belvoir, Va., have grown a 32-lb. thallium chloride crystal for AEC-supported nuclear physics research at the Synchrotron Laboratory of the California Institute of Technology, Pasadena, Calif.

The crystal will provide means of measuring high energy electrons, X-rays, mesons and other atomic components produced by the Synchrotron, or by the penetration of the Synchrotron X-ray beam through targets of various elements.

These particles are directed against the end surface of the crystal and absorbed. Their energies are then converted to emitted light called "Cerenkov Radiation." The intensity of this light is measured quantitatively by electronic instrumentation.

The crystal, 7 in. long and 6 in. in diameter, was grown in a specially designed "furnace" by chemists of the Laboratories' Materials Branch.

The development of materials and growing techniques utilized in the production of large single synthetic crystals of high optical clarity at the Laboratories dates back to World War II, when the Corps of Engineers first became interested in the application of infrared detection systems. Selected crystalline materials furnish an efficient means of energy transmission and dispersion into the far infrared regions.

## NOTES FROM THE NATIONAL BUREAU OF STANDARDS

### IGY WORLD WARNING AGENCY

On July 1, 1957, the National Bureau of Standards radio forecasting center at Fort Belvoir, Va., began serving as the focal point of a world-wide communications network for the International Geophysical Year. From this point, warnings will be flashed to scientists throughout the world to redouble their observational efforts in anticipation of unusual activity in cosmic rays, aurora, earth magnetism, and radio propagation disturbances.

The warnings will be based mainly on world-wide observations of the surface of the sun and on soundings of the ionosphere, the electrically charged upper portion of the atmosphere. When the surface of the sun erupts, shooting out flames for hundreds of thousands of miles, the earth's atmosphere is showered with vastly increased quantities of particles from outer space. This solar bombardment not only causes magnetic compasses to misbehave but also produces brilliant displays of Northern Lights and causes changes in radio communication through its effect on the ionosphere.

The NBS field station was selected as the IGY World Warning Agency by the U. S. National Committee for the IGY. Organized by the National Academy of Sciences—National Research Council, the U. S. National Committee is planning and directing U. S. participation in the IGY under the chairmanship of Dr. Joseph Kaplan. The world-wide warning network is under the general direction of Alan H. Shapley of the NBS Boulder (Colo.) Laboratories, who is serving as Vice Chairman of the United States National Committee for the IGY and international coordinator of IGY communications. The IGY World Warning Agency at Fort Belvoir is headed by Roger C. Moore of the NBS staff.

In addition to the NBS station at Fort Belvoir, the international network includes the radio teletype network of the World Meteorological Organization, virtually all of the commercial communications facilities throughout the world, government facilities (such as military channels and in the United States, the Civil Aeronautics Administration), and special messages broadcast by stations WWV and WWVH (on the NBS radio propagation forecast channels) and their counterparts in other countries. This elaborate and far-reaching network has been set up so that IGY scientists, no matter how remote the site of their work—from Arctic outposts to Pacific islands or the Himalayas—can conduct their experiments simultaneously. Since January 1957 the

warning system has been undergoing a week of advance trials each month.

The International Geophysical Year of 1957–58 has been planned as a massive, coordinated assault by the scientists of the world upon the mysteries of this planet. From July 1, 1957 through December 31, 1958, several thousand scientists representing over 50 nations will make simultaneous world-wide measurements of the earth's interior, its crust, its oceans, its atmosphere, and its immediate cosmic environment. The results of this international enterprise should help answer questions as to the size and shape of the earth, how and where weather is generated, whether the world will continue to grow warmer, why the pull of gravity varies over the earth, the origins of earthquakes, and the causes of radio black-outs.

Because it is not economically feasible for scientists to make intensive world-wide observations every day during the IGY, a series of Regular World Days has been selected in advance for more detailed simultaneous observations. These Regular World Days will be supplemented from time to time by two types of warnings—Alerts and Special World Intervals—which the IGY World Warning Agency will issue when major solar-terrestrial disturbances are expected.

Alerts will be used to notify IGY scientists that a Special World Interval may be called in a few days. The Special World Interval will be called on about eight hours' notice when there is a strong possibility that a major solar-terrestrial disturbance will begin within 24 hours after the start of the interval. The interval will end when the disturbance subsides, or in about 24 hours should the predicted disturbance not materialize.

IGY programs in ionospheric physics, geomagnetism, solar activity, cosmic rays, and aurora will be intensified during Special World Intervals. Some special cosmic ray balloon flights and rocket launching may be made during these periods, with experiments on a stand-by basis, awaiting notification of special conditions from the world warning center.

By 1600 GMT (Greenwich Mean Time) each day, scientists at the World Warning Agency will decide whether to call an Alert or Special World Interval for 0001 GMT the following day or not. This decision will be made with the advice of ionospheric and solar observatories and communications forecasting centers both in the United States and abroad.

If solar conditions justify calling an Alert or Special World Interval, the World Warning Agency will issue messages to Regional Warning Centers in The Netherlands, France, Germany, Japan, and the USSR, and then to Associate Warning Centers in Australia, Antarctica, and Alaska. From these centers, the warnings will be flashed to every IGY field station throughout the world.

The forecasting center in Virginia will also serve as the Western Hemisphere Regional Warning Center. In the United States itself, messages will be put on the U. S. Weather Bureau communications system, so that all U. S. Weather Bureau stations will be alerted. It is then the task of each Weather Bureau station to inform all the other IGY field stations in its locality. The special radio propagation transmitting stations—WWV and WWVH in the United States, LOL in Argentina, and JJD in Japan—are expected to serve as a secondary method of informing the world IGY stations of Alerts and Special World Intervals.

### POWDERS FOR LOW-TEMPERATURE INSULATION

The Cryogenic Engineering Laboratory of the National Bureau of Standards, Boulder (Colo.) Laboratories, has undertaken to improve the Dewar vessel with the aim of developing methods that are adaptable to large-scale equipment. This project is one aspect of a broad program devoted to developing and improving cryogenic techniques and obtaining engineering data.

One of the most important problems in low-temperature technology is that of keeping heat out of low-temperature equipment. The problem of thermal isolation from the external surroundings was largely solved by Sir James Dewar, who contributed one of the most important inventions to cryogenic technique—the vacuum-jacketed container, with walls of high reflectivity for rejecting thermal radiation.

The best of dewars—as these containers are called—has a heat leakage by radiation across its vacuum jacket of the order of 1 milliwatt/cm.$^2$ between walls with temperatures of 300° K. and 76° K. The leakage by thermal radiation between walls at 76° K. and 20° or 4° K. in the same vacuum-walled flask is of the order of 2 microwatts/cm.$^2$ for the very best flasks, and usually runs from 10 to 20 microwatts/cm.$^2$. If the gas pressure is allowed to increase, all the above "heat leaks" are increased correspondingly.

For large-scale apparatus, investigation has indicated that evacuated powders such as perlite, diamataceous earth, and silica aerogel provide a decided improvement over the standard vacuum-walled low-emissivity Dewar vessel. In addition, the evacuated-powder insulated vessel can have high-emissivity walls and a thousand times greater gas pressure without destroying its usefulness as an efficient low-temperature vessel.

A study of powder insulation has been undertaken by M. M. Fulk, R. J. Devereaux, and J. S. Schrodt of the Bureau's Cryogenic Engineering Laboratory to obtain additional data on powders already in use and to determine the properties of powders and mixtures of powders that have not previously been investigated.

Reduction of the over-all heat leak through powders can be achieved in a number of different ways that may be used either separately or in

combination. The total heat transport is lowered by pumping out the gases, by increasing the degree of subdivision, by increasing or decreasing the density of the powders, by adding metallic powders to make the material "opaque" to thermal radiation, by disrupting or minimizing the amount of crystal structure, and possibly by other methods which remain to be explored.

By using the methods now available, one can make the over-all thermal resistance of a vessel with a 2- to 3-cm. thickness of some evacuated powders as good as or better than a "high-vacuum" jacketed vessel in the temperature interval of 300° to 76° K.

At present, single-component evacuated powders find their principal commercial application in vessels for large-scale storage and transportation of liquid oxygen. A further step has been the construction of a powder-insulated vessel for truck transportation and short-time storage of liquid hydrogen, the first large-scale liquid hydrogen vessel which does not require use of a thermal shield cooled by an auxiliary refrigerant such as liquid nitrogen. The superior performance of the metal powder mixtures permits extensions of these applications and should encourage the use of this insulating principle in other applications requiring efficient insulation. Possible fields of application are gas liquefaction and rectification and low-temperature chemical processes.

# THE FRANKLIN INSTITUTE

## COMMITTEE ON SCIENCE AND THE ARTS

*(Abstract of Proceedings of Stated Meeting held Wednesday, June 12, 1957.)*

HALL OF THE COMMITTEE,
PHILADELPHIA, JUNE 12, 1957.

MR. JOHN H. NEHER in the Chair.

The following reports were presented for final action:

No. 3222: Work of Robert Watson-Watt on the Development of Pulsed Radar in the United Kingdom.

This report recommended the award of an Elliott Cresson Medal to Robert Alexander Watson-Watt, of Thornhill, Ontario, Canada, "For his conception of pulsed radar for the vital air defense of Great Britain, for his solutions of important technical problems, and for his inspiring leadership which resulted in the timely development of these radar systems."

No. 3322: The Franklin Medal.

This report recommended the award of The Franklin Medal to Hugh Stott Taylor, of Princeton, New Jersey, "In recognition of his many notable contributions to the science of physical chemistry, particularly his development of the widely significant theory of heterogeneous catalysis, and his eminent achievements as author, editor and teacher."

No. 3327: Henderson Medal.

This report recommended the award of the George R. Henderson Medal to the Association of American Railroads, "In recognition of the many achievements of the Mechanical and Engineering Divisions in the many fields of railway engineering such as the standardization of cmaterials and equipment; the development of the tightlock coupler, the standard passenger ar axle, and the solid journal bearings."

D. S. FAHRNEY,
*Secretary to Committee*

## LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor. 205 items have been added the past month.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Mondays, Tuesdays, Thursdays, and Fridays; 2 P.M. until 10 P.M. on Wednesdays, and 9 A.M. until Noon on Saturdays.

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## NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, DIRECTOR

### CAREER FORUM

At the time of its inception The Franklin Institute was greatly concerned with the instruction of young people in subjects related to the useful arts. There was an urgent need for instructed workers in the infant industries, but no facilities existed for educating willing students until the Institute opened its classes to young and old. As the industries expanded, educational facilities had to be created upon a scale far beyond the capacity of any private institution. As technical schools and colleges were developed to meet the growing needs and our Institute relinquished its self-imposed responsibilities, the interest in the young generation as the potential reservoir to fill vacancies in the technical industries remained unchanged. The only change was in the methods employed.

The creation of the large scientific and technical museum was one method of filling a need whereby continued education was assured after formal education had ended. Meanwhile, an alert watch was kept for other opportunities to serve young people in furthering their technical knowledge. Recently, a new field of endeavor presented itself.

For the past few years it has been apparent that the requirements for newly created industries could not be met by the normal flow of applicants for employment. Some methods of stimulation had to be applied which would ensure that a larger percentage of students were prepared to fill the ever increasing demands of the scientific and technical occupations.

Two years ago, with the co-operation of the American Society for Metals, the Institute's Museum staff sponsored a "Career Forum" in which it was proposed to display to high school students the opportunities open to them for employment in the metals industry. Counsellors were to be provided to guide the students in the choice of studies that would fit them for work as metallurgists, literature was provided, and a lecture-demonstration opened up a few of the more interesting aspects of the work. When this career forum was announced through the schools, the response was embarrassing. So many students applied for admission to the sessions that immediate steps had to be taken to repeat them during the following month.

The Institute was by no means the only agency seeking to encourage likely students to prepare for a technical education. In fact so many agencies were at work that the student was enabled to pick and choose

which he would attend. It was gratifying to learn that the Institute's effort had been rewarded by an enthusiastic response.

The measure of success attained by this first effort justified duplicating it on behalf of another industry. Electronics as a career was made the subject of the second forum, with the co-operation of the Philadelphia Section of the Institute of Radio Engineers. A program was arranged for November, 1955, which was believed would avoid the the embarrassment created by the response to the first forum but, again, the response was overwhelming, and arrangements had to be made hastily to extend the program over a third day.

With the method now established further forums were arranged in association with the American Chemical Society and the American Institute of Chemical Engineers. The first of these, held on November 14, 1956, was directed at junior high-school students, and repeated on April 12, 1957, for those at senior high-school level. Two sessions had to be arranged at each of these forums to accommodate the large number of interested students.

A student counselling day was arranged by the Museum with the aid of Engineering and Technical Societies Council, which was designed to offer guidance to those contemplating an engineering career.

At each of these forums a conspicuous feature has been the attendance of counsellors from the industries who have placed their knowledge and experience at the service of students who were seeking guidance in choosing a career.

The undoubted success of this form of student guidance has encouraged the Museum staff to arrange for further career forums in which students will be introduced to the opportunities offered by other industries and they will be advised how their technical education can be best designed to fit them for the careers they choose to follow.

THE FRANKLIN INSTITUTE LABORATORIES
FOR RESEARCH AND DEVELOPMENT
NICOL H. SMITH, DIRECTOR

# MAGNETOSTRICTION MEASUREMENTS ON SOME ORDERED AND DISORDERED Fe-Pt ALLOYS *

BY

E. KLOKHOLM AND F. J. DONAHOE

As a continuation of a general study of the relationship between magnetic properties and the ordering process in Fe-Pt alloys, the longitudinal magnetostriction for alloys containing 24.2, 27.1, and 30 per cent[1] platinum in the ordered and disordered states has been measured. Measurements of the dependence of magnetostriction on the degree of order in this phase region are complicated by structural changes. Within the composition region for which these alloys order, there exists a martensitic type phase transformation from a high temperature face-centered cubic structure (designated as $\gamma$-phase) to a low temperature body-centered cubic structure (designated as $\alpha$-phase). The transition temperatures depend markedly on the degree of order. For example, the transition temperature of the 24.2 per cent alloy ranges from 300° K. for a quenched sample to 78° K. for a slowly cooled specimen. The $\gamma$-phase on ordering becomes stable at room temperature. An ordered $\alpha$-phase has not been observed. The $\alpha$-phase is always ferromagnetic, while the Curie temperature of the $\gamma$-phase depends on the degree of ordering (**1**, **2**).

In order to clarify the effect of the structural modifications on the magnetostriction of the ordered and disordered alloys, the magnetostriction was measured with the samples at room temperature, then at liquid nitrogen temperature, and finally after returning to room temperature. The data are summarized in the table below. The correlation with the structural changes was satisfactory.

In general, the magnitude of the longitudinal magnetostriction of the $\gamma$-phase alloys increased on ordering; this is in agreement with the data of Akulov *et al.* (**3**), but not with results of Kussmann and von Rittberg (**4**).

* This work was performed under the cooperatively sponsored Magnetics Research Program at The Franklin Institute Laboratories for Research and Development. This is an excerpt from a paper of the same title, which was presented at the AIEE Conference on Magnetism and Magnetic Materials, October, 1956.

[1] All alloy compositions are specified as atomic percentages.

*The Longitudinal Magnetostriction at Magnetic Saturation* ($\lambda \times 10^6$).

| Alloy %(Pt) | Quenched from 1000° C. Measured at: Room Temp. | 80° K. | Room Temp. after 80° K. | Ordered Measured at: Room Temp. | 80° K. | Room Temp. after 80° K. |
|---|---|---|---|---|---|---|
| 24.2 | 25.3 | 10 | 60 | 58 | 270 | 40.3 |
| 27.1 | 29.7 | 10.3 | 26.6 | 73.5 | 108 | 57 |
| 30.0 | 78 | 10 | 76 | 77 | 167 | 80 |

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THE BIOCHEMICAL RESEARCH FOUNDATION
ELLICE McDONALD, FOUNDER
WILLIAM G. BATT, DIRECTOR

# GLUCOSONE. I. PREPARATION AND PROPERTIES *

BY

MARIE T. HUDSON AND GLADYS E. WOODWARD

D-Glucosone has been prepared by various investigators as a syrup or as a vitreous or amorphous solid in varying degrees of purity. In connection with studies in this laboratory on glucose analogs as inhibitors of carbohydrate metabolism, a pure preparation of D-glucosone was desired. This report describes the preparation of glucosone as a solid, a method for its purification, and some properties of the purified material.

## PREPARATION

The method of preparation was basically that of Brüll (1) in which glucose phenylosazone is split with pyruvic acid. Modifications as described by Topper and Lipton (2) and by Dorfman *et al.* (3) have been incorporated into the method.

The glucose phenylosazone was recrystallized several times from ethanol since Smith (4) stated that the purity of an osone depended on the purity of the starting osazone. The pyruvic acid was triply distilled before use, and the apparatus was fitted together through glass joints wherever possible to avoid contamination through rubber connections.

Glucose phenylosazone (3 gm.), pyruvic acid (3 ml.), and 40 per cent iso-propyl alcohol (150 ml.) were placed in a 500 ml. round-bottomed flask. A reflux condenser was attached to the flask and carbon dioxide gas was introduced into the reaction mixture through a glass tube in the condenser. The mixture was refluxed on a water bath for 30 min. at which time everything was in solution. The iso-propyl alcohol was then distilled off under reduced pressure in an atmosphere of nitrogen gas. With the removal of the alcohol the unreacted glucosazone and the pyruvic phenylhydrazone formed in the reaction, precipitated. The solution was cooled in an ice bath to complete the precipitation. The precipitate was removed by filtration and washed with 30 ml. of water. The filtrate was extracted five times with 60 ml. benzene to remove unreacted pyruvic acid. The aqueous fraction was heated to 80° C. in an atmosphere of carbon dioxide gas and then poured through a well washed charcoal (Darco G-60) mat several times until it was water clear. The liquid was evaporated, under reduced pressure in an atmosphere of nitrogen gas, to a light

* Taken in part from a thesis submitted by Marie T. Hudson to the Faculty of the University of Delaware in partial fulfillment of the requirements for the degree of Master of Arts, June 1956.

amber syrup. The syrup was triturated with 15 ml. of chloroform-ethanol (3:2) until a white solid formed. The color remained in the liquid phase. The material was collected by centrifugation and the solid washed by centrifugation several times with the same solvent mixture. The solid was placed in a vacuum desiccator over sulfuric acid to dry and the desiccator was kept under refrigeration. A yield of approximately 150 mg. was obtained. It is believed that the low yield was due to the adsorption of glucosone on the charcoal since it was later found that charcoal had quite an affinity for glucosone. The material appeared to be slightly hygroscopic so was kept in vacuum over calcium chloride. The material was kept in the refrigerator although it is not certain that this precaution was necessary.

*Purification*

The adsorption of glucosone on charcoal was used as a basis of further purification of the compound by means of column chromatography. A pyrex glass tube, 20 mm. in diameter and 50 cm. long, drawn out at one end was used as the column. Pyrex glass wool was pushed into the narrow end of the tube. Gentle suction was applied throughout the following procedure. A mixture of 40 gm. Darco G-60 and 20 gm. Celite 535 in water was poured into the column in small portions. When the column was finally packed, it was washed with 100 ml. of 25 per cent ethanol followed by 100 ml. of water. The column was then washed with 100 ml. of 1 per cent hydrochloric acid to dissolve any mineral material that may have been present with the charcoal. The column was finally rinsed with 350 ml. water. The adsorptive capacity of the column was found to be about 800 mg. of glucosone. The glucosone, combined from several preparations, was added to the column as a water solution and eluted with 15 per cent ethanol. The eluate was collected in 50 ml. aliquots. The fractions were tested for the presence of glucosone by means of diphenylamine trichloroacetate. This reagent had been used as a spray by Hough *et al.* (5) for the chromatographic detection of sugars. About 0.04 ml. of each aliquot was spotted on Whatman No. 1 filter paper, sprayed with the diphenylamine reagent and heated in an oven (100°–105° C.) for 5 min. The presence of glucosone was indicated by a purplish blue ring or spot. Fractions containing glucosone were evaporated to dryness under reduced pressure in an atmosphere of nitrogen gas. A flaky vitreous mass was obtained which was transferred to a vacuum desiccator over calcium chloride for further drying.

## PROPERTIES

*Reducing Properties*

An aqueous solution of the glucosone reduced Fehling's reagent in the cold. The glucosone also gave a reducing value approximately 50

per cent of the reducing value for glucose with the Hagedorn-Jensen (6) reagent for sugar determination. Whether the sample was dried under vacuum in an Abderhalden pistol drier over phosphorus pentoxide (refluxing ethyl ether or methyl alcohol) or dried in a vacuum desiccator over calcium chloride, approximately the same reducing value was obtained for the glucosone with the Hagedorn-Jensen method. The same reducing value was also obtained with the colorimetric technique of Nelson (7) in combination with the improved copper reagent of Somogyi (8). Bayne and Fewster (9) reported that their glucosone possessed approximately 40 per cent of the reducing power of glucose by means of the above Nelson-Somogyi procedure.

*Precipitation with 2,4-Dinitrophenylhydrazine*

Dixon and Harrison (10) found that equal volumes of an 0.03 per cent solution of glucosone and 2,4-dinitrophenylhydrazine reagent gave a precipitate in periods within 1 hr. at 37° C. With our preparation of glucosone 1 ml. of an 0.01 per cent solution mixed with an equal volume of the 2,4-dinitrophenylhydrazine reagent at 37° C. gave in 2 min. a cloudy solution and in 12 min. a definite precipitate.

*Paper Chromatographic Behavior*

Paper chromatography indicated that the glucosone was homogeneous. The ascending method was used for the development of the chromatogram with the top layer of *n*-butanol, pyridine, benzene, water mixture (5:3:1:3) as the solvent. On Whatman No. 1 filter paper in 4 hr. at room temperature, the glucosone moved to an area centered at $1\frac{1}{2}$ in. from the base line, while the solvent front moved $6\frac{1}{4}$ in. from the base line. The glucosone was detected as a purplish blue spot with the diphenylamine trichloroacetate spray (5).

A former glucosone sample studied in this laboratory (11) has now been found not to be homogeneous when examined by this chromatographic method. The glucosone sample contained a second, faster moving component. This second component moved to an area which extended $2\frac{1}{4}$ to $3\frac{3}{4}$ in. from the base line and appeared as an orange spot when sprayed with the diphenylamine reagent. It was not possible to distinguish this second component with the alkaline potassium permanganate or the ammoniacal silver nitrate reagents which had been used by others (3, 12) to show homogeneity.

It was also shown by the diphenylamine reagent that less pure glucosone samples were obtained when the refluxing time of the glucosazone and the pyruvic acid was extended for more than 30 min., during the original preparation of the glucosone. When a reaction mixture was refluxed for $1\frac{1}{2}$ hr. and chromatographic tests made on samples taken every $\frac{1}{2}$ hr., the glucosone spot developed with approximately the

same intensity in all samples, but the orange spot showed an increase in intensity with increase in refluxing time. A refluxing time of longer than 30 min., therefore, did not seem to increase the yield and on the other hand gave a less pure product.

*Melting Point*

To our knowledge no melting point for glucosone has ever been reported in the literature. Before purification, our material softened at 80°–87° C. and decomposed over a range to 115° C. The purified sample softened and decomposed over a range 85°–96° C. The observations were made on samples in a capillary tube and with a slow rate of heating.

*Optical Rotation*

To observe the rotation of glucosone 0.04 gm. of material were dissolved in 1 ml. of water and placed in a capillary 1-dm. polarimeter tube. The light source was the D line of sodium. The initial reading was taken with the polarimeter 6 min. after the solution was made and subsequent readings were taken at time intervals as indicated in Table I. It was found that this glucosone gave an observed specific

TABLE I.—*Optical Rotation of D-Glucosone in Aqueous Solution.* ($c = 4.0$, $\ell = 1$, $t = 29°$ C., D line of sodium)

| Elapsed Time hours:minutes | Observed Rotation | Specific Rotation |
|---|---|---|
| 0:06 | −0.95° | −23.75° |
| 0:20 | −0.74 | −18.50 |
| 0:30 | −0.67 | −16.75 |
| 1:00 | −0.50 | −12.50 |
| 1:30 | −0.39 | − 9.75 |
| 2:00 | −0.31 | − 7.75 |
| 2:30 | −0.23 | − 5.75 |
| 20:00 | 0.00 | 0.00 |
| 25:36 | 0.00 | 0.00 |

rotation which changed from −23.75° at 6 min. to 0.00° within 20 hr. at 29° C. No further change was noted in intervals from 20 to 168 hr. Becker and May (**13**) reported that the specific rotation of their glucosone was −2.5° and −1.4° after 20 and 55 min., respectively. Bayne, Collie, and Fewster (**14**) observed a specific rotation change of −10.6° to +7.9° in 275 hr.

*Absorption Spectrum*

The ultraviolet absorption spectrum of this preparation of glucosone presents a complex picture. The observations were made in cells of 1-cm. light path in a Beckman model DU spectrophotometer.

Glucosone in neutral or acid solution had been reported to show a maximum in the absorption curve at 277 mμ (9) or 297.5 mμ (15) and a minimum at 244–251 mμ (9, 15). The absorption curve determined with our preparation in aqueous solution did not show such a maximum or minimum. When the glucosone was dissolved in distilled water at a concentration of 1 mg. per ml. (pH about 4.5) and the absorption determined immediately, the curve was as shown in Fig. 1. There was very little absorption in the region of 360 to 270 mμ. The absorption increased slightly as the wavelength was decreased in this region, but below 270 mμ there was a sharp continuous increase in

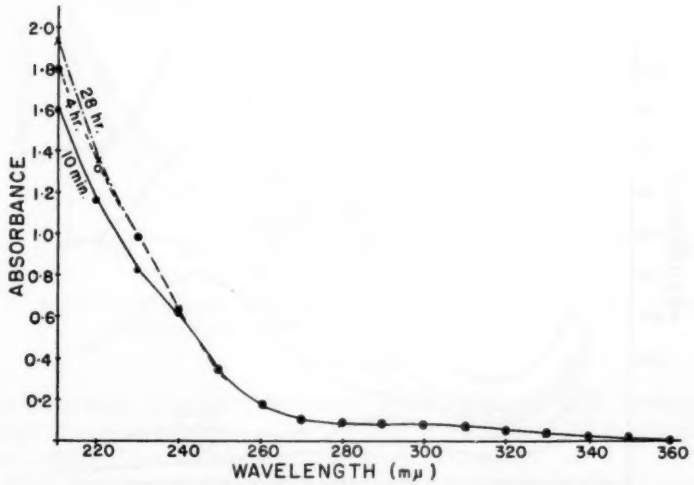


FIG. 1. Ultraviolet absorption spectrum of D-glucosone (1 mg./ml.) in distilled water at different time intervals after dissolving.

absorption as the wavelength was decreased further. There was a slight increase in the slope of the curve below 240 mμ when the curve was redetermined at 4 hr. and at 28 hr. after the solution was made. When the glucosone solution was acidified further with hydrochloric acid at a concentration of 0.01 *N*, the absorption at wavelengths below 240 mμ was slightly lower, but the shape of the curve was unchanged.

It had been reported that glucosone in alkaline solution gave a maximum in the absorption curve at 310 mμ (16) or 318 mμ (9) and a minimum at 245 mμ (9) or 249 mμ (17). With our material, this picture was shown immediately after adding sodium hydroxide solution to an aqueous solution of the glucosone to produce a concentration of 0.25 mg. glucosone per ml. of 0.25 *N* sodium hydroxide, but the absorption which was maximal at 315 mμ rapidly decreased. The changes which took place in the absorption spectrum are shown in Fig. 2.

After 4 hr. the peak at 315 mμ had almost disappeared and a second, although smaller, peak had developed at 270 mμ. After 48 hr. the peak at 315 mμ had completely disappeared. The minimum at 245 mμ which was apparent in the earliest curve persisted throughout, although the absorption gradually decreased over the whole range of wavelengths.

With solutions made to contain 0.1 mg. of glucosone per ml. of 0.1 *N* or 0.25 *N* sodium hydroxide, the peak at 315 mμ was hardly apparent. The peak at 315 mμ was obtained, however, with glucosone

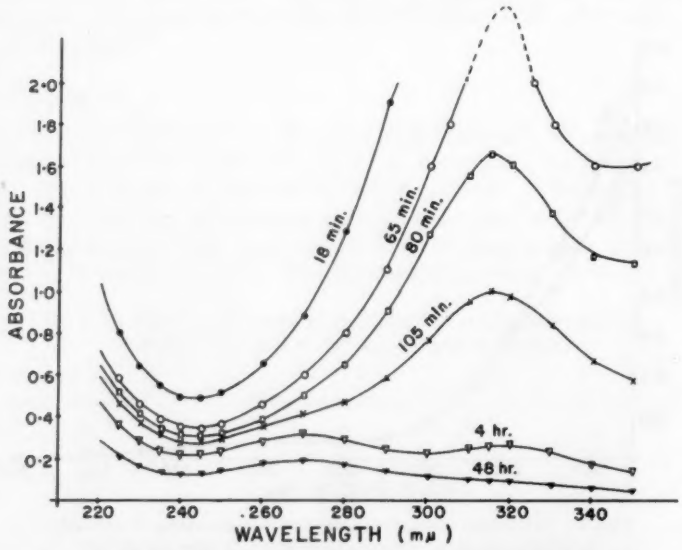


FIG. 2. Ultraviolet absorption spectrum of D-glucosone (0.25 mg./ml.) in 0.25 *N* NaOH at different time intervals after making alkaline

at a concentration of 0.25 mg. per ml. in 0.05 *N* sodium carbonate. In this case, the peak was slow in developing. There was an induction period of about 40 minutes during which there was even a slight decrease in the absorption at 315 mμ, but at this time the absorption started to increase very rapidly and reached its height at about 55 minutes. The height was considerably less than that obtained with this concentration of glucosone in 0.25 *N* sodium hydroxide (Fig. 2). After 55 min., changes in the absorption of the carbonate solution of the glucosone then took place as in the sodium hydroxide solution.

Glucosone dissolved in 0.1 *M* phosphate buffer of pH 7.0 at a concentration of 0.5 mg. per ml. gave the absorption curves shown in Fig. 3. Immediately after dissolving the glucosone in the buffer, the

absorption curve showed a plateau at 250–270 mμ which suggested the presence of material which would give a peak at about 265 mμ under certain conditions. The plateau gradually faded out with time, however, as shown by the curves determined at later time intervals.

Since glucosone may exist in several isomeric forms (**9, 13**) the complex picture presented by the absorption spectra determined under different conditions of solution, may be due to shifts in the molecular form of the glucosone.

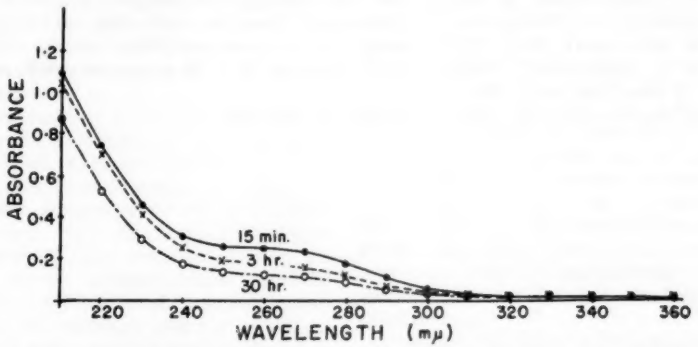


FIG. 3. Ultraviolet absorption spectrum of D-glucosone (0.5 mg./ml.) in 0.1 *M* phosphate buffer, pH 7.0, at different time intervals after dissolving.

SUMMARY

D-Glucosone has been prepared and further purified by charcoal column chromatography. The product appeared to be homogeneous when analyzed by means of paper chromatography.

This glucosone gave a reducing value approximately 50 per cent of that of glucose in both the Hagedorn-Jensen and the Nelson-Somogyi procedures; melted with decomposition over a range from 85° to 96° C.; and mutarotated from an observed specific rotation of −23.75° to 0.00°.

The ultraviolet absorption spectrum of the glucosone in neutral or in acid solution displayed little absorption in the region from 360 to 270 mμ, but a sharp continuous increase in absorption was shown as the wavelength decreased. In alkaline solution the glucosone displayed a maximum at 315 mμ which rapidly decreased with time and a minimum at 245 mμ which was constant; with the disappearance of the peak at 315 mμ a second smaller peak at 270 mμ developed in the absorption curve of this purified glucosone.

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THE CALCULATION OF ATOMIC STRUCTURES, by Douglas R. Hartree. 181 pages, 6 × 9 in. New York, John Wiley & Sons, Inc., 1957. Price, $5.00.

This volume is based on a series of lectures presented while the author was Philips Visitor at Haverford College in the spring of 1955 and later given at Princeton University. The material it contains consists in large measure of the original contributions of the author, who has been a leader in this field from the very beginning of quantum theory. The primary emphasis of the book is on the practical problems encountered in the calculation of the energy, wave functions, and electronic charge distribution of atomic structures. Enough of the basic theory is included to provide the necessary background for the calculational procedures. Very often in this field accounts are given of approximate solutions to the many-electron problem without carefully pointing out the nature of the approximation, leaving the impression with the uninitiated reader that the treatment is exact. Professor Hartree has taken pains to critically evaluate wherever possible all approximate procedures. The need for approximation in this field is vividly shown when one considers that a table of the exact ground state wave function for the Fe atom would require about $10^{83}$ entries, and that such a table would consist of more matter than is contained in the entire solar system!

After an introductory section in quantum mechanics, the variational principle is discussed in detail and applied to the ground state of many-electron atomic systems. The radial wave equations resulting from the central field approximation are derived with exchange for the case where the electron configuration consists of complete groups. A detailed description of the numerical procedures needed for their solution is presented and applied to the equations with and without exchange. The author's extensive experience with these techniques is invaluable in describing the best ways of making initial estimates, starting the solutions for numerical integration, and the subsequent cycling of successive approximations until "input" and "output" fields are equal and the self-consistent field has been obtained. Frequent examples are shown to illustrate the application of the general methods described. Slater's theory of complex spectra is reviewed for the purpose of pointing out the additional complications introduced in calculations on atomic configurations consisting of incomplete electron groups. The variation of atomic wave functions and fields with atomic number is empirically analyzed, giving much of the author's insight into the problem of obtaining starting estimates for new calculations. Knowledge of this variation also allows the calculation of properties of atoms for which a self-consistent field calculation does not exist. Other topics covered are contributions to the total energy from complete and incomplete groups, calculation of relativistic effects, and better approximations to the many-electron atomic problem. The appendices include a partial bibliography of calculations of atomic structures, tables of representative radial wave functions, and recent developments added in proof.

A complete tabulation of all references to calculated atomic structures, rather than the abbreviated list given in Appendix I, would tend to make the book more self-contained. Aside from this one minor criticism, the book is excellent in all respects. Its clarity and orderliness are unsurpassed by any book in this general field. The community of physicists and students of physics will enthusiastically welcome this contribution to their literature which only Professor D. R. Hartree could so uniquely make.

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PHILOSOPHY OF SCIENCE, by Philipp Frank. 394 pages, diagrams, 5½ × 8½ in. New York, Prentice-Hall, Inc., 1957. Price, $5.75.

In the preface to this book, the author points to a prevailing conflict of views. On the one hand we have a responsible group of people urging a halt to scientific advances until social sciences have come abreast of them, while on the other hand we have another group, no less responsible, pleading for more scientists to prevent the nation from lagging behind others that are our political and economic rivals. He devotes this vol-

ume to a thought-provoking study of the philosophy of science as a means of reconciling these conflicting views.

The deeper we dig into science the more its links with philosophy are obvious. The highly specialized nature of scientific training tends to obscure this fact and to make pragmatic results all-important, so that the technical worker comes to regard with suspicion, if not contempt, the man who seeks to establish scientific concepts on a philosophical basis. Science teaching in our schools of higher learning has, for the most part, ignored the philosophical interest.

Dr. Frank, a former professor of Theoretical Physics in the University of Prague, is now a visiting professor at the Massachusetts Institute of Technology. He combines the qualities of scientist and philosopher, so that his book is impartial and objective in viewpoint. This viewpoint affords him an exceedingly broad aspect of his subject and he approaches it with a comprehensive knowledge which permits him to perceive relationships in ideas that are not ordinarily associated. Writing with a refreshing clarity, he marshals the material from the general history of ideas with effect, and supplements this with original contributions in the field that can hardly fail to stimulate the reader to further thought.

In view of the great interest, recently originated at Harvard, of promoting an intelligent "understanding" of science, as opposed to the acquisition of scientific information, this book forms a valuable contribution to a fuller appreciation of science. It is not exclusively a theoretical work. Theories abound, but they are well demonstrated by many pertinent physical proofs drawn from a vast store of personal knowledge.

T. COULSON
*The Franklin Institute Museum*

ORGANIC SYNTHESIS, Vol. I and II, by Vartkes Migrdichian. 1822 pages, 6 × 9 in. New York, Reinhold Publishing Corp., 1957. Price, $35.00.

It is difficult to realize that this monumental work is the achievement of one man. In preparing this treatise on organic synthesis Dr. Migrdichian, a Senior Research Chemist at American Cyanamid Company, has shown that he is thoroughly conversant with the literature of organic chemistry, and, of great importance, that he has carried out many of the synthetic operations himself.

All the subject matter of the two volumes is covered under two main headings, Open Chain Compounds, and Carbocyclic Compounds, The first part covers the whole first volume and one-third of the second volume. The second part covers the balance of the second volume.

Under Part 1, Open-Chain Compounds, the first section, which includes the whole of the first volume, is devoted to saturated compounds. The arrangement, which follows conventional lines, gives the usual reactions of hydrocarbons, alcohols, esters, ethers, and related compounds, with many interesting and unusual comments on individual reactions. In three chapters carbonyl compounds get well-merited attention. Then follows a chapter on acids. Esters and ester condensations are well-handled, as are acid halides, amides and nitriles. Amines, halogen compounds, and the Grignard reaction are covered thoroughly. There are good chapters on the Friedel-Crafts reaction, aliphatic nitro and nitroso compounds, sulfonic and sulfinic acids, and related compounds. Volume I closes with two chapters on organo metallic compounds. In Volume II the study of Open-Chain Compounds continues, under chapter headings of olefinic compounds, unsaturated compounds with functional groups, acetylenic compounds, and the diene synthesis.

In Part 2, Carbocyclic Compounds, the alicyclic compounds are considered, with emphasis on the hydrocarbons. The balance of the volume is devoted to aromatic compounds. The hydrocarbons, phenols, quinones, carbonyl compounds, carboxylic acids, amines, diazo compounds, halogen compounds, nitro and nitroso compounds, and sulfonic acids and related thio- compounds are included.

Separately, the various chapters and sections make interesting and valuable reading for the synthetic organic chemist. It seems to the reviewer that this is the way the books will be and should be employed. None but the most dedicated chemist will start this work on page 3 and finish at page 1734. Nor should it be so. This is a reference work. It is not written for the novice, but for the working scientist who needs definite advice and

guidance on the intricate and often discouraging path of his research.

Too much emphasis cannot be put on the very excellent reference sections which follow each chapter. They are useful and thorough. Also, it should be pointed out here that the work is not yet complete, and that most of the reactions leading to the formation of heterocyclic bodies do not appear. A third volume to cover this missing information is contemplated.

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*American Chemical Paint Co.*

THE AIRGLOW AND THE AURORAE, edited by E. B. Armstrong and A. Dalgarno. 420 pages, diagrams, 7¼ × 9¾ in. London and New York, Pergamon Press, n.d. Price, $22.50.

The most concentrated assault ever attempted on a scientific program began on July 1, 1957. With the inauguration of the International Geophysical Year, thousands of scientists from 56 countries will make the earth a tremendous laboratory. Of highest priority will be the investigation of the atmosphere. It is this atmosphere which filters, reflects, alters and attenuates radiations coming to the earth from outer space. It is these radiations reacting with the atmosphere which gives rise to the airglow and the aurora.

When the satellite program has been fully developed, the scientist will acquire a definitive knowledge of extra-terrestrial radiations. Once these data are available, then the radiations giving rise to the "lights" in the sky can be studied. However, until such time, scientists must learn of these radiations from their effects on the atmosphere. New and powerful devices and abstruse theoretical studies have been brought to bear upon these effects to give the scientists a clue to the radiations and the mechanism of stimulation.

In *The Airglow and the Aurorae* are contained the proceedings of a conference held at the Queen's University of Belfast in late August and September of 1955. In their preface, the editors say, "The papers presented constituted a coherent survey of the most recent work on observational, experimental and theoretical aspects of the phenomena (airglow and aurorae). . . ." This remark is quite appropriate, for the content of the work fulfills the prefatory promise.

The book has eight sections, and the extent of the problem is completely covered in the first by Dr. Sydney Chapman. In his introduction, he covers the achievements and prospects in aurorae and airglow research. His all-too-short history presents a back drop against which the modern work can be assessed.

Sections II and III deal with the physical observations of the airglow and the aurorae while Sections IV and V deal with the theoretical considerations of the airglow and the aurorae.

The airglow is made up of many sources of light—the gegenschein, sodium emission, the zodiacal light, noctilucent clouds, and even starlight contribute to the light of the night sky. For some of these we have adequate descriptions and explanations; for others we can speculate and hypothesize. In Section V, Dr. Joseph Chamberlain presents a paper on the discharge theory of auroral rays. This paper pictures clearly the difficulties in attempting an explanation for the observed phenomena. The explanations, properly called discussions, portray the highly speculative character of this work. Many of the present results are based on assumptions which may be erroneous.

In this same section, the excitation processes giving rise to the aurorae are discussed.

To complement observational results, Section VI deals with laboratory studies of excitation phenomena. Section VII covers the theoretical studies giving use to the auroral lines, and the final section is devoted to the instrumentation necessary to photograph and analyze the airglow and aurora.

*The Airglow and the Aurorae* is the most recent work of its type and the last word in these researches. Until a more definitive work appears, this is the "bible" for workers in this field.

I. M. LEVITT
*The Fels Planetarium*

THE PLASTIC METHODS OF STRUCTURAL ANALYSIS, by B. G. Neal. 353 pages, diagrams, 5½ × 8½ in. New York, John Wiley & Sons, Inc., 1956. Price, $7.50.

Generally, structures have always been designed based upon the elastic properties of a material. But in recent years there has been some agitation to discard this method and, instead, to use the plastic properties. In the classical theory of bending as applied to a

beam of rectangular cross section, when the extreme fibers of the cross section reach their elastic limit, the beam is assumed to carry no further load. In the theory of bending based upon the plastic limit, the beam is assumed to carry load until not only the extreme fibers have reached their elastic limit but also until all the fibers in the cross section have reached this limit. Proponents of the plastic theory claim that the elastic theory is too conservative, that structures will in fact sustain more load than that allowed by it. Tests have substantiated much of the plastic theory of bending and in fact there is much to recommend its adoption. For one who designs structures to the limit of the applied loads based upon elasticity, it is reassuring to know that there is a little extra material to take the loads in case his assumptions weren't quite right; at proof test time he won't perspire quite as much.

This English book on the plastic method of structural analysis (written by a professor of civil engineering in the University College of Swansea), while not broad in scope, thoroughly covers its subject as applied to simple and redundant beams and simple and compound frame bents. The basic hypothesis of the plastic method—which is simply that a plastic hinge forms at points of maximum moments such that the beam rotates at the hinge with no increase in the plastic moment—is described in the opening chapters. The general methods of plastic design—procedure, trial and error method, plastic moment distribution and plastic collapse loads—are then explained. In many structures the criterion for acceptability is often deflection rather than strength, so that it is necessary to know how to compute deflections in this plastic method of design. Professor Neal shows how these are estimated and also how to design for the minimum structural weight. The book is concluded with a discussion of the effect of variable repeated loadings on structures designed according to the plastic theory.

The explanations of the various plastic methods of structural analysis are very thorough; in fact, in some cases it becomes repetitious. Problems with answers at the end of each chapter further amplify the theory. There is a short appendix in which the plastic theory is applied to trusses.

E. W. HAMMER
*The Budd Company*

SCIENTIFIC INFERENCE, by Sir Harold Jeffreys. Second edition, 236 pages, 5½ × 8½ in. New York, Cambridge University Press, 1957. Price, $4.75.

The over-all plan of this book might be described as an attempt to answer the question "How good is the inductive method as applied to discovering the laws of physical science?" It is the author's contention that many inferences which are based on empirical data and which are not made with certainty are, nonetheless, believed by many to be the result of deduction and established with certainty. An interesting discussion between a Botanist and a Logician lends weight to the author's contention and sets the stage for the remainder of the work. At the conclusion of the first chapter, the reader will readily admit that "there is an element of uncertainty in all inferences of the inductive kind," that is, inferences from past observations to future ones.

In support of the method of induction the author states, in essence: Scientific progress is not made by deducing laws of physics and then looking for something to satisfy them. Indeed, mechanics struggled for centuries to reconcile motion with the rule that velocity is proportional to force. Scientific progress is a method of successive approximation in which the scientist states his laws in the most general and simple form possible (based on observation) and then modifies them as later experience indicates.

Starting with the postulate that "it is possible to learn from experience and to make inferences from it beyond the data directly known by sensation," Sir Harold adopts the position that "scientific method can be understood if and only if a theory of epistemological probability is provided." Having taken this stand, he then proceeds to generate a theory of probability with emphasis on the limits and validity of *a posteriori* knowledge. At this point of the book considerable discussion is given to Bayes' theorem and the principal of inverse probability, which, while essential to a general theory of induction, "nevertheless . . . seems to fill many people with terror." In spite of the lengthy discussion allotted this principle, Sir Harold has not, in the reviewer's opinion, done much to remove the shroud of mystery surrounding Bayes' theorem. After reading *Scientific Inference*,

those who had been terror stricken by inverse probability will continue at least equally as terror stricken.

Quite naturally a chapter on sampling and tests of significance follow the discussion on probability. Here we find developed the binomial, normal, Poisson and chi-square laws of distribution.

The fact that exact agreement between physical laws and observation has never been attained has prompted the author to devote a section to the important problem of analyzing residuals and their properties. His analysis indicates that the problem receives a satisfactory answer from the theory of probability. Sir Harold comments that physicists hardly ever give a statement of uncertainty derived from their observations in the required way and that if they would spend a few extra hours their results could become of full use to other people.

The remainder of the book is concerned with some of the chief laws of physics and the observational evidence that supports them. In the large these laws are drawn from Mensuration, Newtonian Dynamics, Light & Relativity and Statistical Mechanics and Quantum Theory. The topics herein discussed are rich in lore and are heavily punctuated with source references.

Though the book would have been complete without it, the author has undertaken in one chapter some of the philosophical problems associated with scientific principles as seen through the concepts of Realism, Idealism, Phenomenalism, Russell & Whiteheadism, Determinism, Nothing but-ism, Materialism, Psychology & Psychoanalysis. He allows that the principal sources of confusion are three: (1) The belief that scientific laws are statements made with certainty (certainty fallacy), (2) The belief that physical measures are exact (exactness fallacy), and (3) The belief that there is a clearly marked boundary between science and ordinary thought (base degrees fallacy).

This is not a book for casual reading—it must be studied thoroughly. Indeed, certain passages must be read and re-read several times in order to determine just what the author is for or against. So many topics are embraced in the space of 236 pages that the over-all treatment is necessarily cursory, if if not desultory. It is the reviewer's opinion that the author could have made his point with less confusion—perhaps he did in the first edition.

LAURENCE C. MCGINN
*The Franklin Institute Laboratories*

TRANSMISSION CIRCUITS, by E. M. Williams and J. B. Woodford, Jr. 156 pages, diagrams, 6 × 9 in. New York, The Macmillan Co., 1957. Price, $4.25.

Professors Williams and Woodford of the Carnegie Institute of Technology have prepared this brief text for a senior electrical engineering course at that school. However, it is difficult to see how a course based upon the contents of this book would fit into the electrical engineering curriculum in the senior year. Its brevity and scattered scope prompt this comment.

As the title implies, the text is related to electrical transmission networks, but these are scattered between power systems and communications applications. It would seem that by the senior year more comprehensive courses in both communications techniques and power transmission would have covered the material presented here.

It might appear therefore that this text would be more appropriately given to non-electrical engineering students as a broadening experience rather than as specialized training. An interesting selection of particular subjects has been made and they have been presented with classical skill. The development of circle diagrams and Smith charts should prove to be interesting and a valuable exercise in special techniques for any engineering student. Since the text itself is short, there is ample opportunity for the instructor to round out the course with material of his own which emphasizes any one of several areas he may feel important to his particular students.

Within the 150 pages the authors have selected a few key concepts pertaining to lumped and distributed-constant transmission lines for both low and high frequency service. The theory of generalized transmission circuits giving voltage and current distributions is presented. With this the determination of transmission line paramaters, including skin-effect, is developed. The appendix contains a brief summary of the solution of transmission circuit problems by means of electromagnetic field theory.

It is difficult to comment upon the significance of a rather standard text of this type. It was produced by the authors to meet their specific teaching need and will probably not be of interest to professionals in engineering who selectively acquire books which offer new and specialized information.

C. W. HARGENS
*The Franklin Institute Laboratories*

THE FIRST 150 YEARS. A HISTORY OF JOHN WILEY AND SONS, INC. 242 pages, illustrations, 7½ × 10 in. New York, John Wiley & Sons, Inc., 1957. Price, $7.50.

We may select our books for any one of a number of different reasons, but it is rare that a reader will choose a book because of its publisher. Since the publisher loses money if a book does not have a successful sale, it stands to reason that no publishing firm could remain in existence for one hundred and fifty years unless its staff exercised a wise choice in the manuscripts selected for publishing. John Wiley & Sons is such a firm. Their record for longevity is a tribute to their business integrity as well as their wisdom.

Today, the firm is distinguished for the publication of technical books, but it was founded at a time when there was little demand for this type of literature. It began when Charles Wiley opened a business in New York as a bookseller, printer, and publisher. The majority of the books he issued bore the names of British authors, but they formed an impressive list, including Byron, Thomas Moore, Ruskin, and Carlyle. Gradually he turned to the sponsorship of American authors like Fenimore Cooper, Margaret Fuller, Melville, Hawthorne, and Poe. After the Civil War, a new fertile field opened for technical literature and the firm seized the opportunity to fill the need.

How well they have accomplished this task may be seen from this present volume. Members of the firm tell the inside story, but authorities in a variety of fields in pure and applied science tell how their scientific literature has been enriched by the firm's contributions. A very progressive firm has paused for a moment to cast a backward glance over its progress and to note the milestones it has erected, marking achievements in scientific thought and attainment. There seems no reason to doubt that there will be many more, and we anticipate adding many volumes bearing the firm's imprint to our library.

Justly proud of the position they have gained for themselves in the publishing world, the firm has told its story in a handsome volume which we cordially commend to the readers of technical books who have reason to be grateful to John Wiley & Sons for the many authoritative works they have made accessible to the public.

T. COULSON
*The Franklin Institute Museum*

ION EXCHANGE AND ITS APPLICATIONS. 173 pages, illustrations, 8½ × 11 in. London, Society of Chemical Industry; New York, The Macmillan Co., 1956. Price, $7.50.

Collected in this volume are the papers read at the conference on ion exchange held at London University in April, 1954. After each of the papers, a transcript of the discussion is presented.

The seven sessions of the conference are divided among four main topics. The first topic presented covers the theoretical aspects of ion exchange and, for the general interest reader, probably represents the most interesting portion of the book. The papers of this section are reviews and critiques of the ion exchange process to date.

The papers covering the second topic: "Industrial Applications," deal largely with ion exchange as a water purification process. It was somewhat disappointing to find no mention of commercial separation processes in which the main interest is the material removed from the water; however, in the two following sections, the deficiency is remedied. In the sections covering analytical and inorganic applications, discussions are given for separation of metals, and the paper by J. M. Hutcheon of Harwell, entitled "Continuous Ion Exchange," proves to be very interesting as a review of various processes.

The concluding sections covering organic and biochemical applications were of particular interest for the papers covering separation techniques for amino acids, nucleic acid degradation products, and proteins.

This book should prove to be of interest to those desiring a well rounded view of the art of ion exchange for either inorganic or organic applications. It would be further desirable as an addition to the library of the general interest reader largely because of the section

covering equilibria and kinetics of ion exchange and the structure and properties of ion exchange resins. As do most symposia, this book fills the need for a review of an expanding field. The regrettable part is the time lapse between the time the conference was held and the arrival of the book on the American scene.

RICHARD HOLLINGER
*The Franklin Institute Laboratories*

AUTOMATION IN BUSINESS AND INDUSTRY, edited by Eugene M. Grabbe. 611 pages, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1957. Price, $10.00.

Modern engineering is becoming more and more concerned with the development of large and complex systems for business, industry and military uses. Automatic control devices and automatic data-processing computers of great flexibility are essential ingredients of such systems.

In order to facilitate the flow of information between management and engineering as well as between the various fields of applied engineering, the Department of Engineering of the University of California arranged a lecture course presented by nationally prominent engineers and scientists from both business and industry. The course was designed to review for engineers and managment from business and industry the present status of developments and applications in the field of automation.

These lectures are published in this book form so that the wealth of information contained therein has been make available to a much larger audience. Each author, and there are a total of twenty-one, has reviewed the automation progress in his particular field with emphasis on new developments, techniques and applications. The treatment of the diversified subjects shows how the fields of feedback control theory, instrumentation, analog and digital computation, and data processing have become integrated as automation is applied to a broad range of control systems. Considerable attention is devoted to electronics and computers as applied to complex control functions and data processing.

The first three chapters discuss, in rather general terms, the fundamentals of automation. An excellent basic glossary of terminology is presented and serves as a useful reference for terms used throughout the book. Chapter 4 presents a thorough exposition of the nature of feedback control systems as applicable to the automation problem. Chapter 5 explores the present status of two basic industrial devices, instruments and automatic controllers. Tools and techniques as well as the measurement of industrial process control variables are discussed.

The next five chapters are concerned with an extensive presentation of the components of electronic computing systems. This mass of material concentrates heavily on descriptive information involving all types of computers and auxiliary equipment. It is an excellent reference source for anyone interested in a thorough yet fairly concise summary of computing system features and capabilities.

An extensive analysis of the fundamental nature of data processing is included in this section. This includes such topics as the economic justification for data-processing equipment, the economics of electronic computation, the speed and capacity requirements of business problems, a comparison of the basic nature of scientific and business problems, an explanation of programming techniques, the interdependence of all features of data processing systems, reliability and error control and the savings which result from the use of automatic electronic data-processing equipment.

A summary and description of analog-to-digital conversion unit and input-output equipment concludes this section. The balance of the book is concerned with the discussion of various types of automatic systems. Chapter 11 describes typical applications in the business data processing field. The growth of office automation is outlined and detailed examples include such operations as parts control, billing and accounting. Chapter 12 involves the development of automatic flight control techniques and such system components as sensing, actuating and amplifying elements.

The automatic production of electronic equipment receives considerable attention. Equipment and techniques are explained and examples of automatic assembly systems are presented. The Project Tinkertoy ac-

tivity of the Bureau of Aeronautics of the U. S. Navy is described.

Several chapters are devoted to the fundamentals of process control in the petroleum and chemical industries and the use of analog computers in industrial control systems. The measurement of operational variables, several types of control systems and servo techniques are treated.

The features of simulation computers in industry are explained through the description of several examples of industrial applications. Chapter 16 presents the features of digital controls for machine tools and the concluding chapters are concerned with the areas of application of automation in manufacturing, the economics of plant automation, and a general discussion of the future of automation in military electronics, business and industry.

Needless to say, this series of lectures attempts to bite off a sizeable chunk when it attempts to describe the many-faceted aspects of such a dynamic subject as automation. The growth of the various fields which form the basis for the development of techniques of automatization has been so explosive that the task of painting one big picture showing the interrelationship of these fields cannot be described as simple. The compilation of this book is one of the first efforts to attempt this difficult task and the opinion of this review is that it does an excellent job of describing the present status of automation. It is probably the most comprehensive work yet published on this subject. Though the book could hardly give complete accounts of the multiplicity of subjects it attempted to cover, it nevertheless contains mountains of information for anyone interested in getting a broad picture of the field of automation. It includes many examples, charts, block diagrams and tables in support of its presentations.

Because each topic covered in the book is presented by an expert in that particular field, none of them suffers serious neglect. One result of this approach, however, is a rather wide assortment of writing styles. This does not detract, appreciably, from the effectiveness of the book.

A. V. PACCHIOLI
*The Franklin Institute Laboratories*

THE LOGNORMAL DISTRIBUTION, by J. Aitchison and J. A. C. Brown. 176 pages, diagrams, 6 × 9¼ in. New York, Cambridge University Press, 1957. Price, $6.50.

In 1903, Kapteyn developed an extensive system of frequency distributions by hypothesizing that a certain function of the random variable, rather than the random variable itself, was normally distributed. This development enabled the ready theorizing of causal relations generating this function which was distributed in a Gaussian fashion, reduced any skew curve to a normal curve, and allowed the convenient fitting of constants to the observationally based curves. Of all the transformations Kapteyn considered, the logarithmic one was most fruitful. This little studied distribution arises in what at first seem to be most unlikely circumstances. One generating principle for the distribution is: when large numbers of items are classified on a selected homogeneity principle, the variate defining the number of items in each class is defined in an approximately lognormal fashion. The major example arises when ores are crushed in successive steps, for then the lognormal distribution describes the size or weight distribution of the fragmental ores. There are innumerable biological, sociological, philological and econometric applications of this little used distribution. In fact, this reviewer is reminded of the generation of this distribution from the estimates made by a large number of passersby of the number of jelly beans heaped up in a store window! There are doubtless many other cases in the psychological literature which would be more meaningfully interpreted with this technique.

The text presents a detailed account of the properties of this distribution, its genesis, estimation problems, and suggested computational techniques. Since the bias of the book is economic, income distributions and consumers behavior are discussed in detail. The major virtue of the book is that it is the sole convenient compilation of the known characteristics of the lognormal distribution and as such is very valuable to technical men confronted with the problem of explaining data having a statistical basis.

EZRA S. KRENDEL
*The Franklin Institute Laboratories*

## BOOK NOTES

INTRODUCTION TO ORGANIC CHEMISTRY, by Louis F. Fieser and Mary Fieser. 613 pages, diagrams, 7 × 9 in. Boston, D. C. Heath and Company, 1957. Price, $7.00.

This new text by the Fiesers replaces their "Organic Chemistry" (1944) and the "Textbook of Organic Chemistry" (1950). The name has been changed to emphasize that the new work is considerably different both in scope and in organization. The same lucid style is evident, however. One of the new features is the inclusion of a section entitled "Applications to Research," in which the authors discuss the history of the research leading to our present knowledge of specific processes, compounds, etc. The marginal notations in boldface type should be a great help to students.

MANUFACTURING ORGANIZATION AND MANAGEMENT, by Harold T. Amrine, John A. Ritchey and Oliver S. Hulley. 454 pages, illustrations, 6 × 9 in. New York, Prentice-Hall, Inc., 1957. Price, $6.75.

The purpose of this book is to present the principles and functions of manufacturing management in a nontechnical way. Aimed at the inexperienced person in the management field, it is suitable for engineers and non-engineers. Topics covered include: Modern Manufacturing Management; Financial Management; Industrial Organization; Marketing Management; Research and Development; Plant Location and Buildings; Design of Manufacturing Processes; Motion and Time Study; Material Handling; Purchasing; Quality Control; Wage and Salary Administration.

VECTOR ANALYSIS, by Louis Brand. 282 pages, 6 × 9 in. New York, John Wiley & Sons, Inc., 1957. Price, $6.00.

Based partially on the author's previous work "Vector and Tensor Analysis," this new book is more elementary in its approach than the former one. The fundamental theory of vector analysis is presented in clear language and is illustrated by examples and numerous problems. It has been written for mathematicians, physicists and engineers who are using vector methods in their work.

MECHANICS FOR ENGINEERS, by Ferdinand P. Beer and E. Russell Johnston, Jr. 697 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1957. Price, $8.00.

This new text has been prepared by two Lehigh University professors for the standard college course in statics and dynamics. The usual subjects are covered, each topic beiug presented in three parts, namely, new material, sample problems, and problems for assignment. The book is well printed and well bound.

THE MICROBIAL WORLD, by Roger Y. Stanier, Michael Doudoroff and Edward A. Adelberg. 682 pages, illustrations, 6 × 9 in. New York, Prentice-Hall, Inc., 1957. Price, $8.00.

The authors, who are all professors of bacteriology at the University of California, have prepared this text for beginners in microbiology who have a general knowledge of biological fact and theory. Part I on "The Properties of Microorganisms" describes the discovery of and the methods for studying microorganisms; Part II—"The Ecology of Microorganisms"—covers their distribution in nature, their influence on their environment and their use by man. A third short section, entitled "The Biological Background," has been included to provide the reader with a summary of biological concepts and to show him where microbiology fits into the over-all picture. One of the unusual features of the book is the placement of an annotated bibliography of 19 titles at the front of the book. This should be a useful and interesting text for teacher and student.

THEORIE UND TECHNIK DER PULSMODULATION, by E. Hölzler and H. Holzwarth. 505 pages, diagrams, 6¼ × 9¼ in. Berlin, Springer-Verlag, 1957. Price, not given.

This German work covers AM, FM, PAM, POM, PPM, and PCM and combinations of these, in detail. Six main sections are divided into several chapters each, which are subdivided for clarity. A very complete table of contents lists all of these subdivisions. Treated in detail are: frequency modulation and time (multiplex) transmission; spectrum analysis; scanning theory; quantizing; coding; network theory; echo; band frequency requirements; square wave generators; sawtooth and step oscillation; time filters; noise; and circuits. An appendix supplies additional mathematical derivations, wiring diagrams and graphs for the specialists. An extensive bibliography and a modest index complete the volume. This carefully prepared and well illustrated volume should prove to be a valuable reference work for anyone working in the frequency modulation field.

FLOTATION, by A. M. Gaudin. Second edition, 573 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1957. Price, $12.50.

The second edition of this outstanding book on the flotation process will be welcomed by those who have been using the first edition (1932) as their "bible." The changes in the art of flotation over the last 25 years have been so great that the book has been completely rewritten and most of the illustrations are new. The second edition includes new material in the fields of crystal chemistry, surface chemistry, fluid mechanics and solution chemistry. The author has also rearranged his material with the intention of stimulating interest in scientific and technical problems which are still to be solved in the flotation process.

THE ENJOYMENT OF MATHEMATICS, by Hans Rademacher and Otto Toeplitz. 204 pages, diagrams, 6 × 9¼ in. Princeton, Princeton University Press, 1957. Price, $4.50.

Originally published in German in 1930, this English version is a translation of the 1933 German edition, with two additional chapters by Herbert Zuckerman.

The "enjoyment" of mathematics to be gained from reading this book is not limited to mathematicians—although the titles of the chapters might scare off some of the amateurs! Anyone with a good solid foundation in high school math can follow the ideas presented. The book cannot be classified a "popular" mathematics book, however, for it emphasizes fundamental principles, not games and tricks.

The authors have tried to break down the "high wall" which surrounds mathematicians, by explaining the intrinsic values, not by showing specific uses. Each of the 28 chapters is an independent unit; each, if mastered, will give the reader a fuller appreciation of the science of mathematics.

EARLY ELECTRICAL MACHINES, by Bern Dibner. 57 pages, illustrations, 8¼ × 10¼ in. Norwalk (Conn.), Burndy Library, 1957. Price, $1.50 (paper).

The subtitle of this beautifully printed monograph aptly describes its contents—"The Experiments and Apparatus of Two Enquiring Centuries (1600 to 1800) That Led to the Triumphs of the Electrical Age." The book has no formal chapter divisions; instead, boldface heads set off separate sections of the text. Sample "headings" include: Robert Boyle, The Leyden Jar, Benjamin Franklin, The Van Marum Machine. These sections are well written and delightfully illustrated with reproductions of old drawings, portraits, paintings, photographs, etc. Anyone interested in electricity will be well rewarded for the time spent in reading this excellent addition to the history of science books.

## PUBLICATIONS RECEIVED

The Editors wish to call attention to these new books, received in the JOURNAL office for review. Lack of space prevents publication of more than this listing.

THE CLIMATE NEAR THE GROUND, by Rudolf Geiger. Second printing, revised, 494 pages, diagrams, 5½ × 8¼ in. Cambridge, Harvard University Press, 1957. Price, $6.00.

RAYONNEMENTS DE PARTICULES ATOMIQUES, ÉLECTRONS ET PHOTONS, by André Berthelot. 192 pages, diagrams, 6¾ × 10 in. Paris, Masson et Cie., 1956. Price, 1800 fr.

PHYSIQUE DE L'ATMOSPHERE. TOME I. PHÉNOMÈNES D'ÉMISSION DANS L'ATMOSPHÈRE, by Etienne Vassy. 338 pages, illustrations, 6 × 9½ in. Paris, Gauthier-Villars, 1956. Price, $10.18 (3500 fr.).

SCIENTIFIC FRENCH, by William N. Locke. 112 pages, 5 × 7 in. New York, John Wiley & Sons, Inc., 1957. Price, $2.25 (paper).

A SHORT DICTIONARY OF MATHEMATICS, by C. H. McDowell. 63 pages, 4¼ × 7 in. New York, Philosophical Library, 1957. Price, $2.75.

THE CONQUEST OF THE ANTARCTIC, by Norman Kemp. 152 pages, plates, 5½ × 8½ in. New York, Philosophical Library, 1957. Price, $4.75.

EPILEPSY, by Letitia Fairfield. 159 pages, 5 × 7½ in. New York, Philosophical Library, 1957. Price, $4.75.

LIGHT, VEGETATION AND CHLOROPHYLL, by J. Terrien, G. Truffaut and J. Carles. 228 pages, diagrams, 4¾ × 7½ in. New York, Philosophical Library, 1957. Price, $6.00.

INTRODUCTION TO CHEMISTRY, by Joseph C. Muhler, Charles S. Rohrer and Ernest E. Campaigne. 452 pages, diagrams, 6 × 9 in. New York, Prentice-Hall, Inc., 1957. Price, $6.75.

ENGINEERING ECONOMY, by H. G. Thuesen. Second edition, 581 pages, diagrams, 5¾ × 8½ in. New York, Prentice-Hall, Inc., 1957. Price, $6.95.

PROCEEDINGS OF THE SECOND RETMA CONFERENCE ON RELIABLE ELECTRICAL CONNECTIONS. 103 pages, illustrations, 8½ × 11 in. New York, Engineering Publishers, 1957. Price, $5.00 (paper).

SCIENTIFIC GERMAN, by George E. Condoyannis. 164 pages, 5 × 6¾ in. New York, John Wiley & Sons, Inc., 1957. Price, $2.50 (paper).

**High Speed Communication Among Data Processing Machines.**—Bell Telephone Laboratories have announced a new experimental device which will enable business machines to "talk" directly with one another at a speed of about 1000 words a minute. Printed material could thus be sent over lines like those used for telephone purposes, but at a speed sixteen times that of conventional teletypewriter systems.

This new device, tentatively called a data subset, is designed to give accurate transmission among wide varieties of information processing machines, such as electronic computers, electric typewriters, or adding machines. Transmission of the data is automatic, and voice communication can precede or follow the operation.

Information is placed on magnetic tape by an electronic computer, electric typewriter or other machine. In contrast to holes punched in paper tape used in many other automatic machines, letters, numbers or symbols are placed on the magnetic tape in the form of magnetized spots. A group of seven such spots identifies the particular character. Some spots are magnetized in such a way that they represent "zero" and others so that they represent "one." All information put on the tape is translated into a code using combinations of these two numbers. The chief advantage of magnetic tape is that its speed is easily adapted to match the speed with which the signals can be sent over telephone wires. It also has the advantage of quiet operation.

After its preparation, the tape is then inserted into the data subset machine. The person on the "sending" end talks by telephone with the person on the "receiving" end to notify him that the message is to be transmitted. The recipient turns on a receiver mechanism and the sender then starts transmission. An automatic signal indicates when the taped message is completed. The person on the receiving end removes the tape from the machine and inserts it in the office machine for decoding. On an electric typewriter, for example, the mechanism could be arranged to type out the message automatically from the magnetic tape. This would be set for the speed of the typewriter and could be at a faster speed than a typist could attain.

To provide essentially error-free transmission, extra error-checking equipment can be added. In the event of a disturbance in transmission, the receiving equipment will then reject the portion of the transmission affected and automatically cause it to be retransmitted.

The present laboratory models are about the size of office typewriters. If they were engineered for maximum compactness, they could be made considerably smaller. The small and convenient size is made possible by the use of transistors, semiconductor diodes and ferroelectric crystals. No vacuum tubes are used. The functions of memory, logic, oscillation, modulation, and amplification, which would ordinarily be performed by vacuum tubes and their associated components, are here performed by the products of solid-state electronics.

By the use of transistors, the machine can be one-twentieth the size

required if vacuum tubes are used. The use of transistors reduces power requirements to about 5 per cent of that necessary to operate a like machine built with vacuum tubes.

Because of its great speed, the equipment uses the information-handling capacity of telephone circuits with high efficiency. In many instances, there is no urgency requiring that a message be sent at 1000 instead of 100 words per minute. But, for a given amount of information, the machine reduces drastically the amount of time the telephone line would need to be used.

Equipment could be set up to function in off-peak telephone traffic hours. For example, the subset could transmit over one line from 5 P.M. until 9 A.M. the following day the equivalent output of some 30 expert typists typing at full speed for 8 hours.

Typewriter preparation of the magnetic tape is only one of many methods which could be employed. More complex business machines would perhaps find even greater use for this type of transmission system. For instance, a bank might employ an electronic computer to compile account numbers, deposits, withdrawls and balances. By placing the data on magnetic tape, the bank could quickly send the information to one or several branch locations simultaneously.

The accuracy and speed of large-scale accounting and bookkeeping procedures have been greatly heightened in recent years by the growing use of modern computing machines. Business, government, and private organizations use them to figure payrolls, conduct inventories, bill customers, or to control the manufacture and distribution of goods.

The efficiency of many of these business practices is limited by the relatively slow methods of sending the information from one machine to another. With the data subset this transportation hurdle is overcome and a transmission system will be available to meet the rapid pace set by electronic data processing.

Most present-day business machines could be modified to produce the required magnetic tape. Tests and careful laboratory studies must continue, Bell Laboratories reported, before this system can be made available for commercial use.

**"Self-Erasing" Mechanical Blackboard.**—A mechanical blackboard that "erases itself" automatically to help its operators find the best of 2 billion billion possible answers to a problem has been developed at The University of Michigan.

Called MITAB—for Michigan Transportation and Assignment Blackboard—the low-priced device can provide short cuts in 20 minutes to answers that would take a big electronic computer years to work out the "long way around," its inventors claim.

Designed and built by mathematicians and engineers in the Operations Research group of the U-M's Engineering Research Institute for about $5000, MITAB cannot properly be called a computer because it cannot remember instructions. It can be directed, however, to add, subtract and perform other simple functions.

According to Robert Machol, ERI systems engineer who participated in MITAB's development, the device has potential as a research and instructional tool. It points the way to solution methods which can then be adapted for even more efficient use of conventional computers.

MITAB's face is made up of 400 indicators much like those used to re-

cord mileage on an automobile, 800 lights and 400 switches. The indicators, each of which registers from zero to ten, are arranged in 20 rows and 20 columns.

If the rows are said to represent the machines available for jobs in a factory, for example, and the columns the various jobs to be done, the indicators can be set to show the cost of each job on each machine.

With 20 jobs and 20 machines, Machol points out, there are 2,432,902,008,176,640,000 possible ways to assign the jobs. The problem is to find the "best arrangement"—the least total cost, the fastest operating time, or some other criterion.

If the problem were to be worked out the "long way," it would require a billion years to solve, even on a high-speed computer, Machol says. On MITAB, however, it is possible to add and subtract from rows and columns and perform other mathematical tricks to arrive at the solution in minutes. Thus "shown the way," a computer could then solve the original problem in seconds.

The mathematical steps could be worked out on a 20-by-20 grid on a blackboard, Machol notes, but the effort involved in erasing, adding, subtracting, identifying elements, changing the order of numbers, and other procedures would be prohibitive. The mechanical blackboard does this automatically as the operator spins a dial the same as that on a telephone.

"MITAB removes the drudgery from such an operation," Machol says, "helping the operator perform steps accurately and rapidly. And when it is finished, it can restore the problem just as it was originally set up."

He says MITAB is applicable to such problems as the determination of the shortest or cheapest routes for a fleet of ships operating around the world, the best assignment of a group of men to a variety of jobs, the most efficient distribution of goods from scattered factories to warehouses in different regions of the country.

As a tool for teaching, the device is especially valuable because students can watch the effects of various mathematical changes as the problem is solved. MITAB is now being used as an instructional aid in the U-M Industrial Engineering Department. ERI researchers also are using MITAB in an attack on the unsolved "traveling salesman problem," involving the shortest route between a number of cities.

The device was suggested by Merrill M. Flood, associate director of ERI. James Munkres discovered, and proved mathematically, a set of rules which invariably leads to efficient and rapid solution of this class of problems.

Engineers Bruce Weinert, Harold Sherman and Joseph Hoagbin built the device, mostly in their spare time with second-hand equipment. Building funds were supplied by the Engineering Research Institute.

**Improved Vigoureux Printing Method.**—An improved technique for vigoureux printing of wool, which will substantially decrease the time needed for dye fixation and increase depth of color achievable, has been perfected by the Commonwealth Scientific and Industrial Research Organization of Australia, Dr. Gerald Laxer, director of the Wool Bureau's Department of Science and Technology, has announced.

The improved method involves no change in the sequence of events and no changes in the machinery used for normal vigoureux printing, Dr. Laxer said. The new method involves the addition of benzyl alcohol or other

suitable solvents to the dye paste at the rate of one half to one gallon of benzyl alcohol to 10 gallons of paste.

This replaces certain unnecessary constituents used in conventional formulae, and no additional costs should therefore be involved, Dr. Laxer stated. By using a suitable solvent and increasing the pressure of saturated steam, it is possible to fix the dye in 5 to 20 minutes, depending on the steam pressure and type of dye used.

Vigoureux printing is done on slivers or slubbing fed through a machine that has a roller with raised bars. The wool is impregnated with the dye when it comes in contact with the revolving bars. After printing, the wool is wrapped with wet covers to prevent the paste from drying out prior to steaming.

Any dye which is more soluble in benzyl alcohol, or in any other solvent used, than in water can be applied with advantage by this method, provided the fastness properties of the dye are satisfactory. Dr. Laxer pointed out that, in general, the properties which confer high wash fastness on a dye also make it soluble in benzyl alcohol.

Premetallized dyes are particularly suitable and are recommended in preference to chrome dyes. However, the somewhat alcohol-insoluble chrome dyes can be used if a levelling agent is added to the paste. Many acid dyes are also suitable.

Saturated steam, available in most mills, at high pressure fixes the dye quickly and acetic acid in the formulae prevents alkali damage. Steaming, under suitable conditions, will give 90–95 per cent dye fixation. It is not economical to fix more than 95 per cent of the dye.

For light shades, steam pressure below 5 psi. should be employed when using premetallized or acid dyes. Saturated steam at a pressure of 10–15 psi. should be used for darker colors. Because different dyes have different rates of fixation, the time of steaming must be adjusted for specific conditions. With the new technique, only 4 minutes are required for 95 per cent fixation with a premetallized dye at a steam pressure of 15 psi.

**New Microwave Ferrites Developed at Bell Laboratories.**—New ferrite materials having properties particularly well adapted for a broad range of microwave applications have been developed by L. G. Van Uitert of Bell Telephone Laboratories. The desirable properties include controllable saturation magnetization, low dielectric loss, and a high degree of reproducibility.

The new ferrite materials constitute a forward step in production of the unique nonreciprocal microwave devices in which solid materials themselves perform complex circuit functions, and signify a new microwave epoch in ferrites.

The new materials are essentially magnesium, manganese, aluminum ferrites or nickel manganese ferrite with a small amount of copper replacing some of the magnesium or nickel. The addition of the proper quantities of copper and manganese to the basic ferrite is advantageous from several points of view. By increasing the reactivity of the mixture, copper decreases the necessary firing temperature by at least 100° C. Under comparable conditions this results in lower porosity and improved uniformity in the fired material. The manganese addition decreases electrical conductivity and thus the dielectric losses in these low porosity materials.

Microwave ferrites with low saturation magnetizations are obtained by

the modification of magnesium ferrite. The saturation magnetization of this ferrite can be decreased in a controlled way by substituting aluminum for a part of the iron. While materials compounded in this fashion are basically satisfactory, their refractory nature makes it difficult to reproduce the magnetic properties required for many microwave applications. The added copper minimizes this difficulty, and also increases slightly the Curie temperature for comparable saturation magnetization.

Microwave measurements on these new ferrites indicate that in general they are comparable to or better than similar materials containing no copper. Applications include microwave gyrators, isolators, and similar devices.

**Color X-Ray Viewer.**—The development of a color X-ray viewer system that increases the readability of X-ray pictures through the use of contrast enhancement and color TV techniques, has been announced by Philco Corporation, of Philadelphia.

The Philco "Exicon" X-ray viewer system performs three essentially different, but equally important, functions to quickly and accurately extract a maximum amount of information from an X-ray transparency. The system enhances X-ray contrast in "gray scale" variations, increases readability by utilizing full color and magnifies an area being viewed.

The Philco "Exicon" X-ray viewer provides electronic contrast enhancement and displays X-ray areas of equal optical density in the same color. Areas having differential optical density are reproduced in other colors such as blue, cyan, green, yellow, orange and red. Thus, opaque solutions injected into the blood system can be followed in color through the heart and blood vessels of the lungs and other organs, lending more accuracy and realism to the X-ray picture. Tissues not clearly seen heretofore will now be made visible in color. The new system consists of monochrome and color monitors, an operators' console and a flying spot scanner. An X-ray negative placed before the flying spot scanner will be magnified and separately reproduced in enhanced monochrome and color. Negatives may be quickly shifted about to obtain magnified images of any portion. The system may be used in conjunction with closed circuit color TV installations.

The eye does a poor job of remembering different shades of gray, just as the ear is forgetful of the intensity of sound. Color, on the other hand, is quite easily remembered. "Exicon," by transforming the gray scales into corresponding varieties of color hue, permits the eye to compare not only adjacent areas of the subject matter, but also separated areas. All areas of equal density in the X-ray negative are displayed in the same color. The system should be invaluable to the medical profession for diagnosis, instruction and demonstration.

Before processing the information contained on an X-ray negative through a contrast-enhancing device and a color converter, the information must be transformed into an electric signal. This is accomplished through the use of a Philco flying spot scanner. The heart of this device is a cathode ray tube. By means of a sharply focused, internally generated, electron beam, a very small spot of high luminous intensity may be produced on the tube's phosphor screen. This bright spot is then focused by a lens system upon the X-ray transparency under examination.

An amount of light, proportional to

the transparency of each small area of the negative, passes through the negative and is picked up by a photo electric tube. There the light is transformed into a video signal which is then amplified. This signal is then fed to a contrast enhancing device in which adjacent areas having a difference in brightness that is hardly discernible may have their relative contrast enhanced well above the threshold of visibility. This TV signal is now fed to a black-and-white monitor and through a color monitor where the X-ray picture is reproduced in color.

On the color monitor, the strongest signal obtained from particularly transparent sections of the X-ray negative will be portrayed as the color red; the weakest signal will show up as blue. Between the extremes, colors range through orange, yellow, chartreuse, green and cyan. The system has been made sufficiently flexible so that different color sequences may be realized. It is also possible to electronically convert the image from a negative to a positive in an instant.

The new Philco "Exicon" viewer has already attracted considerable attention in the medical profession. A prototype model is currently being demonstrated before representatives of a number of major civilian and military medical installations.

Other uses for the new system now under study are: aerial reconnaissance, air traffic control and various industrial techniques such as those employed in the fields of metallurgy, chemistry and pharmacology.

**Rectifier Operates at 1200° F.**—A silicon carbide rectifier has been operated at temperatures up to 1200° F., the General Electric Research Laboratory has announced. Similar devices made with other semiconductor materials, notably sillcon, have been able to operate only up to the 400–500° F. range.

"Although currents of several amperes have been rectified with usable efficiency, the practical future of silicon carbide in rectifiers and other solid-state devices will depend on progress in the preparation of this highly stable but extremely stubborn material," according to Dr. Guy Suits, GE vice president and director of research.

High-temperature semiconductor devices would have an important role in many military and aircraft electronic applications. (GE said that its silicon carbide rectifier also has been operated at temperatures down to as low as 100° F. below zero.)

"Most of the semiconductor devices now on the market have been made from single-element materials—principally germanium and silicon—but future progress appears to depend on learning more about compounds," Dr. Suits said. "A major problem in all semiconductor research, that of achieving extremely pure materials, becomes increasingly difficult when more than one element is involved."

The GE research director emphasized that the silicon carbide rectifier is still in laboratory development.

**Chicks Instead of Monkeys in Polio Vaccine Testing.**—Baby chickens have proven consistent in comparison with monkeys in testing potency of poliomyelitis vaccine, according to results of an 18-month study revealed by Parke, Davis & Company. The use of chicks permits an accurate measure of the vaccine's potency, places potency testing on a quantitative rather than qualitative basis, and greatly reduces chance of error. Additional studies still in progress on use of chicks indicate the time now required

in potency tests (21 days) might be trimmed considerably.

At present, polio vaccine is tested for potency by vaccinating monkeys in the same manner as humans. The production of antibodies of the monkey serums is then measured to determine how effective the vaccine will be when used in humans. Since individual monkeys (12 monkeys are required in testing potency of each batch of polio vaccine) vary widely in their ability to produce antibodies, the test results are often difficult to interpret. In the type of test presently required, the quality of the vaccine is largely determined by the serum antibodies produced. The higher the response, the better the vaccine.

In contrast, young chicks are tested on a quantitative basis, determined by the degree to which the vaccine can be diluted before it fails to produce antibody response. In other words, a given vaccine can be diluted many times and a more absolute measure of its effectiveness obtained.

White leghorn chicks, 7 to 10 days old, used in the tests are unnatural hosts for the polio virus and their antibody-producing mechanisms are more independent of the variation in response found in monkeys. Preliminary and as yet inconclusive studies show the effectiveness of the vaccine in producing antibodies in chicks could be measured in 3 to 5 days after only one inoculation was given. At present, monkeys used for testing receive three inoculations of polio vaccine over a 14-day period. An additional seven days are devoted to determining the vaccine's effectiveness on them.

The over-all studies suggest that the use of chicks presents a practical means of measuring antigenic potency of polio virus preparations. Over and above the obvious cost advantage, the chick test provides a number of advantages over other methods.

**Excavating in Frozen Ground.**—Army Engineers at Fort Belvoir, Va., are investigating the possibilities of excavating frozen ground in the Arctic. Construction equipment specialists at the Engineer Research and Development Laboratories have bored holes into a $4' \times 4' \times 7'$ block of frozen silt and sandy loam with a new truck-mounted earth auger developed by the H. B. Williams Manufacturing Company of Dallas, Texas. The block had been frozen in Fort Belvoir's Climatic Test Laboratory, where for four days the temperatures hit 55 degrees below zero.

Holes 18, 24 and 30 in. in diameter were drilled 4 ft. deep during the tests. High grade alloy cutting teeth, hard-faced with tungsten carbide, were used on the auger.

Difficulties in discharging the spoil were encountered by the auger, which operates on the centrifugal force principle. Studies are underway to develop a more suitable means of dispersing this material. High-speed jets of compressed air are currently being considered. The auger, one of two big units under test at the Engineer Research & Development Laboratories, can dig a 6-ft. diameter hole 20 ft. deep at a rate of a half-foot per minute in ordinary, unfrozen earth. Its "big brother," a trailer-mounted machine also developed by the Williams Company, is capable of digging a hole 9 ft. in diameter to depths of 70 ft.

These augers will be useful in the construction of field fortifications and emplacements, shallow wells, waste disposal pits, and underground storage facilities as well as for other numerous applications.

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